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Between a growth and a fixed mindset: Examining nuances in 3rd-grade students' mathematics intelligence mindsets

Connie Barroso [a,b,*], Colleen M. Ganley [b,c], Robert C. Schoen [c,d], Christopher Schatschneider [b,e]

[a] Department of Educational Psychology, Texas A&M University, College Station, TX, USA
[b] Department of Psychology, Florida State University, Tallahassee, FL, USA
[c] Florida Center for Research in Science, Technology, Engineering, and Mathematics, Learning Systems Institute, Tallahassee, FL, USA
[d] School of Teacher Education, Florida State University, Tallahassee, FL, USA
[e] Florida Center for Reading Research, Florida State University, Tallahassee, FL, USA

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ABSTRACT

Children's growth and fixed intelligence mindsets in mathematics are noted as important sources of mathematics motivation and achievement. Nuanced beliefs about the malleability of mathematics intelligence that lie between fixed and growth mindsets may also be important to consider for children's mathematics learning, yet little is known about whether children endorse these in-between beliefs and how they fit in the popular growth and fixed mindset framework. In this study, we investigated nuanced mindsets in mathematics, which we term "mixed" intelligence mindsets, alongside fixed and growth mindsets in a sample of 698 third-grade students in the United States. Factor analyses using data from a newly developed mathematics intelligence mindset scale indicated good and similar fit of three multidimensional models. Two of these models included mixed mindset items, one with a combined growth and mixed mindset factor and another with a separated mixed mindset factor. Strong positive correlations were found between the growth and mixed mindset factors. Mathematics achievement had a moderate positive correlation with mixed mindset and a moderate negative correlation with fixed mindset. These correlations were both significantly stronger than the small correlation between mathematics achievement and growth mindset. Our findings suggest mathematics intelligence mindset is multidimensional and the addition of a mixed mindset aspect could improve the adequacy and precision of the conceptualization and measurement of the growth mindset dimension. In practice, mixed mindsets may provide teachers and parents with more flexible messages to present to children when encouraging them to engage in adaptive achievement behaviors in mathematics.

Mathematics is one of the most difficult school subjects that students struggle with during primary and middle school (Dündar et al., 2014). These ongoing struggles can have long-term consequences for individual's academic and financial outcomes, such as lower attainment of mathematics proficiency (Tyson & Roksa, 2017) and reduced career prospects and finances (Lazarides et al., 2020; Ritchie & Bates, 2013). Children's mathematics struggles can also lead to societal consequences in the form of a reduced future science, technology, engineering, and mathematics (STEM) workforce (Hinojosa et al., 2016). Mathematics skill interventions are integral for improving mathematics abilities (Williams et al., 2022), yet non-cognitive factors, such as beliefs about ability, can also be essential in encouraging the motivation and persistence that students require to make the most out of these interventions.

Mathematics intelligence mindsets are the beliefs that people have about the malleability of mathematics intelligence. These beliefs develop in childhood and can impact student resilience in the face of struggles and failures in mathematics (Aronson et al., 2002; Blackwell et al., 2007; Dweck, 2008; Stipek & Gralinski, 1996; Yeager et al., 2019).

Growth intelligence mindsets, defined as beliefs that ability is malleable and can be developed, have been linked with adaptive achievement behaviors and motivation that lead to positive achievement outcomes in childhood (Dweck & Leggett, 1988). Fixed intelligence mindsets, defined as beliefs that ability is inherently stable and cannot be changed, are typically maladaptive for children's achievement outcomes (Dweck & Leggett, 1988). Strong mathematical abilities and motivation to learn mathematics are crucial factors for fostering

* Corresponding author at: Texas A&M University, 704 Harrington Tower, College Station, TX 77843, USA.
E-mail address: cbarroso@tamu.edu (C. Barroso).

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student's interest to pursue STEM careers. Despite this, growth mindsets are found to be less frequently held by children in the domain of mathematics than in other subjects (Gunderson et al., 2017; Heyder et al., 2021). Children are also found to have stronger levels of fixed mindsets in mathematics compared to other subjects (Gunderson et al., 2017; Heyder et al., 2021). Unfortunately, greater fixed mindsets about mathematics ability have larger negative impacts on mathematics achievement compared to the impact of other domain-specific mindsets on their respective achievement areas (Gunderson et al., 2017; Heyder et al., 2021; Shively & Ryan, 2013).

These results using the growth and fixed mindset framework suggest that growth mindsets should be nourished and fixed mindsets reduced in order to aide in improving achievement outcomes, especially in mathematics. Although the growth and fixed mindset framework has been the primary framework used to characterize the beliefs people have about the malleability of intelligence, empirical evidence has indicated that intelligence mindsets may be more nuanced than the current fixed and growth distinction primarily used in the field (Boyum, 1988; Droege & Stipek, 1993; Kinlaw & Kurtz-Costes, 2003). For example, research shows that some people do not clearly hold one mindset or the other. Some estimates suggest that anywhere between 15% and 37% of people hold mindsets that fall in between a growth and fixed mindset (Dweck et al., 1995; Kaijanaho & Tirronen, 2018). Given the impact of mindsets on mathematics outcomes and the evident gap in the popular growth and fixed mindset framework used to characterize these beliefs, it is prudent to investigate the potential presence of nuanced beliefs about the malleability of mathematics intelligence. Do children hold nuanced beliefs that overlap between growth and fixed mindsets? If so, how closely related are these nuanced beliefs to growth and fixed mindsets? Further, how do these nuanced beliefs relate to mathematics achievement? In the current study, we investigate these questions and aim to more closely examine children's beliefs about the malleability of mathematics intelligence.

1. Intelligence mindsets in children

Theoretical work in the 1980s proposed the important role that intelligence mindsets play in achievement-related behaviors (Dweck & Leggett, 1988). Correlational and experimental research has further supported many of these theoretical tenets and explored their existence in children, adolescents, and adults. Within the mathematics domain, the majority of mindset research has been conducted on adolescent and young adult samples (Blackwell et al., 2007; Romero et al. 2014; Shively & Ryan, 2013). This literature, along with the available but limited work investigating domain-specific and broad mindsets during early and middle childhood, has been informative in describing the development of intelligence mindsets over time and their importance to achievement outcomes.

Research suggests that children's beliefs about their abilities form as a result of many factors, including environmental- and experiential-related factors such as the influence of role models and the increasing number of self-evaluation experiences that children encounter as they grow up (Haimovitz & Dweck, 2017). Intelligence mindsets adopted early in childhood can change as children get older, though findings about how they change over time are inconsistent. Some research suggests that children's views get more fixed-oriented as they develop (i.e., Dweck & Bempechat, 1983; Stipek & Daniels, 1988; Stipek & Iver, 1989), whereas other research suggests that beliefs become more growth-oriented over time (Burke & Williams, 2009).

Research has linked children and young adolescents' mathematics-specific intelligence mindsets to their learning goals (Blackwell et al., 2007), challenge-seeking behaviors (Rege et al., 2021), future enrollment in advanced mathematics courses (Romero et al. 2014), class engagement (King & Trinidad, 2021), and mathematics achievement (Sisk et al., 2018; Yeager & Dweck, 2020). Even more, research on intelligence mindsets defined broadly with young adolescents finds stronger relations between mindset and achievement for students from lower socioeconomic status backgrounds (Claro et al., 2016) and those who have lower academic performance (Yeager et al., 2019). Some research also finds that the mindset-achievement relation is mediated by motivation variables, such as academic grit (Kaya & Karakoc, 2022), self-efficacy for boys (Huang et al., 2019; Su et al., 2021), and mathematics identity (Cribbs et al., 2021). Other research indicates long-term changes in achievement that are influenced by earlier levels of intelligence mindset. For example, one study found that grade 3–6 students' early fixed mindsets in mathematics led to a greater rate of decline in mathematics achievement over time (McCutchen et al., 2016).

As a result of these important achievement-related associations and influences of mathematics intelligence mindsets, growth mindset interventions have been developed and implemented to capitalize on the benefits of specific types of mindsets. A recent meta-analysis found that the overall average effect of growth mindset interventions on achievement is very small ($d = 0.08$; Sisk et al., 2018). The impact of interventions targeting beliefs about the self can be reduced by various intervention components, such as the intervention duration and number of sessions (DeBacker et al., 2018). The impact of interventions can also vary if individual differences are not taken into account or if there are differences in the social context or delivery method of the intervention (Hanselman et al., 2017). Mindset interventions have nevertheless become mainstream in recent years and are often implemented in mathematics classrooms at the elementary, middle, and high school levels in the United States and around the world (Blackwell et al., 2007; Savvides & Bond, 2021).

2. Conceptualization of mathematics intelligence mindset

Growth mindset interventions have primarily been guided by results using the growth and fixed mindset lens to conceptualize intelligence mindset. However, conducting research that only uses the growth and fixed mindset framework may have potentially egregious implications regarding what is known about the development and impacts of mathematics intelligence mindsets during childhood, particularly if the characterization of the construct is incomplete. Therefore, it is important to study the specificity and breadth of the type of beliefs that children might hold about the malleability of intelligence in the mathematics domain.

The initial conceptualization of mathematics intelligence mindset follows that of foundational research on the originally named construct of "implicit theory of intelligence" (Dweck et al., 1995; Dweck & Leggett, 1988). In this work, intelligence mindset is conceptualized as a unidimensional construct, positing that a child who holds a growth mindset does *not* hold a fixed mindset, and vice-versa. Much of the well-cited empirical research investigating children's intelligence mindsets assumes this unidimensionality in the construct as well. Researchers typically ask participants to rate their level of agreement on three fixed-oriented items (e.g., *Your intelligence is something about you that you can't change very much*) using a 6-point Likert-type scale, with scale anchor points ranging from (1) strongly agree to (6) strongly disagree (Dweck & Henderson, 1989). The arithmetic mean of the raw responses to the three items determines the questionnaire score for each individual, and a continuous variable is thus obtained to represent the construct of intelligence mindset on a spectrum. With this scoring method, children who score at the lower end of the variable are assumed to hold a strong fixed mindset and those who score at the higher end are assumed to hold a strong growth mindset.

Growth mindset items have also been developed and used by reverse-scoring scores on the growth mindset items and averaging the raw scores from the growth- and fixed-oriented items together (Boyum, 1988; Dweck et al., 1995; Leggett, 1985). Some scholars have even conceptualized intelligence mindset as a two-dimensional construct instead of a unidimensional one (i.e., Kinlaw & Kurtz-Costes, 2007). Studies have found empirical support for a two-dimensional model over a

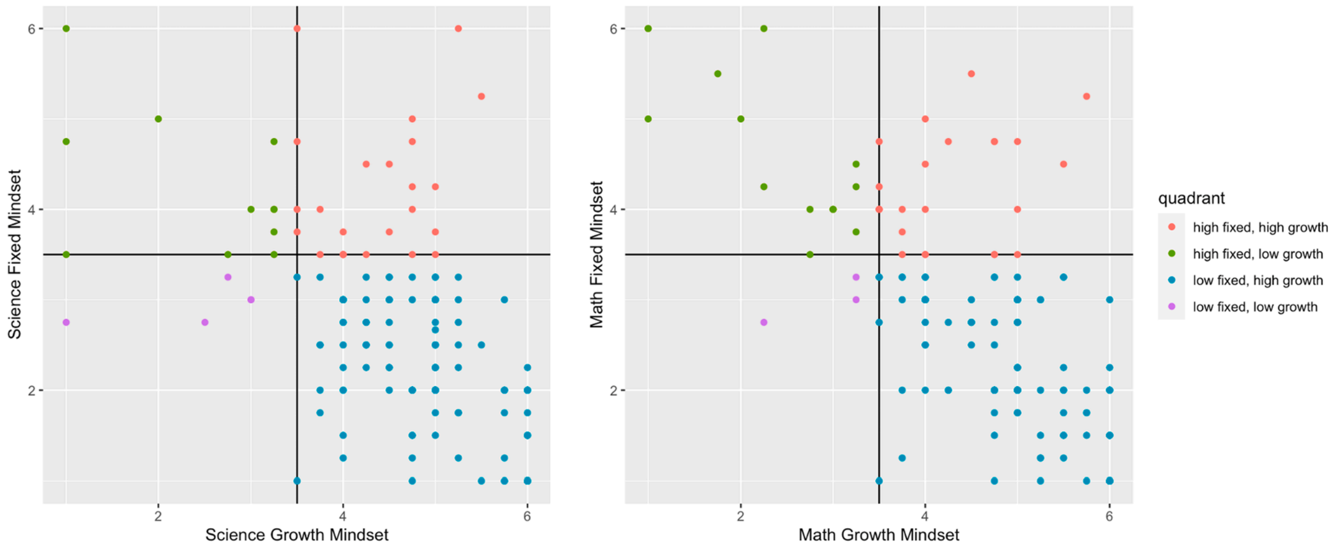

Fig. 1. Scatterplots using data from an unpublished study with a sample of 194 college students (Barroso Garcia, 2016). Total scores from fixed intelligence mindset items are plotted against total scores from growth intelligence mindset items in the domains of science (left) and mathematics (right). Black lines indicate the midpoints of the scales used.

one-dimensional model, with intelligence mindset represented by separate growth and fixed mindset dimensions (Bråten & Strømsø, 2005; Dupeyrat & Mariné, 2005; Kaijanaho & Tirronen, 2018). Importantly, two scores are obtained from this two-dimensional model. The variability across these two scores presents the possibility that a subset of people could simultaneously agree that intelligence is fixed *and* that it is malleable.

There is previous empirical evidence that supports the idea that growth and fixed beliefs can be held simultaneously (Chiu et al., 1997; Dweck & Henderson, 1989; Hong et al., 1999; Levy & Dweck., 1997). For example, we present two scatterplots in Fig. 1 using data from an unpublished study that examined intelligence mindsets in mathematics and science in undergraduate engineering majors (Barroso Garcia, 2016). These figures show participants' average scores on mathematics and science growth mindset scales, separately, plotted against their average scores on mathematics and science fixed mindset scales, respectively. The data show that there are some individuals who report low levels of both growth and fixed mindsets in these school subjects. These students might not believe that either fixed or growth mindset represents their beliefs about the malleability of intelligence. The data also show that some individuals report holding high levels of both of these domain-specific mindsets simultaneously. The beliefs of these individuals may be described as a mixture of both growth and fixed mindsets.

In some studies, researchers have overlooked the latter phenomenon by classifying participants into strict "growth" and "fixed" mindset groups and removing individuals from analyses who reported having overlapping beliefs (i.e., Dweck et al., 1995). However, early descriptions of the construct of intelligence mindset support the possibility that a person might have overlapping growth and fixed beliefs, suggesting that:

> …perhaps the most appropriate view represents an integration of both entity [fixed] and incremental [growth] theories, that is, a recognition of present differences in relative ability but an emphasis on individual growth in ability (see also Nicholls, 1984). (Dweck & Leggett, 1988, p. 263)

3. Nuanced beliefs about the malleability of intelligence

Understandably, it is believable that nuanced beliefs about the malleability of intelligence could be held by adults who have developed cognitive abilities enabling them to be flexible in their thinking. In fact, the research described in the previous section regarding individuals who hold overlapping growth and fixed mindsets is primarily obtained from studies analyzing data from adults. But other research evidence also suggests that children in middle childhood could have the ability to hold and understand nuanced beliefs about the nature of intelligence. Around age 8, children's understanding of people's traits and characteristics begin to develop from mostly concrete or categorical descriptions of observable features a person has (e.g., gender, hair color, height) to more nuanced, dispositional descriptions of traits that a person has (i.e., intelligence, personality; Gonzalez et al., 2010). Children in middle childhood also begin to compare their own abilities in academic subjects with those of their peers who they spend a majority of time with in the classroom (Marsh et al., 2015). These experiences could initiate student's flexible thinking and inform the development of nuanced beliefs about how malleable a person's abilities are relative to others in the world.

The phenomenon of flexible thinking and nuanced beliefs may similarly transfer to the beliefs that people have about the malleability of intelligence. The popular mindset framework depicting beliefs about the malleability of intelligence as either growth- or fixed-oriented may not adequately capture the nuances in beliefs held by people that overlap between growth and fixed mindsets. With a focus on children's intelligence mindsets within the mathematics domain, it is prudent to closely examine these overlapping, nuanced beliefs children might have. Importantly, existing research can be leveraged to inform how children's nuanced beliefs about the malleability of mathematics intelligence could be defined and measured.

3.1. Beliefs based on the capacity for intelligence to change over time

Growth mindset has previously been described as the belief of "not yet", which reflects the *capacity* of intelligence to change *over time* (Dweck, 2008). Early versions of intelligence mindset scales included items that assessed the belief that intelligence can be different in the future compared to the present (i.e., *You can be more intelligent in the future; You can be more intelligent in the future than you are now*; Boyum, 1988). Recent research on differences in beliefs children have about their own abilities as children versus that of adults also suggests that children think differently about the capacity to change *present* intelligence than the capacity to change *future* intelligence (Gunderson et al., 2017). Early research on beliefs about the malleability of intelligence had previously included the belief about the "stability of intelligence" (e.g., Droege & Stipek, 1993). Yet, these beliefs about stability are not measured in the intelligence mindset scales that are widely used today (e.g., Dweck et al., 1995; Yeager et al. 2019).

3.2. *Beliefs about individual differences in the malleability of intelligence*

Nuanced mindsets that lie between growth- and fixed-oriented conceptions could also be characterized as a "recognition of present differences in relative ability but an emphasis on individual growth in ability" (Dweck & Leggett, 1988). In other words, these beliefs could consist of the belief that every person can change their intelligence (i.e., growth mindset), with the acknowledgement that there are individual differences regarding either the present level of ability or change in ability (akin to fixed mindset in that these individual differences are fixed – even if everyone grows, those starting higher in ability will still be higher). In line with this idea, a person might believe that the "starting" level of intelligence could be different between people and/or that the rate of change in intelligence could differ between people. For example, a person may think that everyone can learn mathematics, but that for some people it will be easier than it is for others, suggesting that growth has some nuance to how it works across individuals.

4. Current gaps in intelligence mindset research

Some past research that has been done to elucidate nuances in beliefs about the malleability of abilities has not been widely used to inform the study of this construct in recent years. For example, empirical research by Pomerantz and Ruble (1997) described multiple dimensions of conceptions of ability and studied the implications of these conceptions on self-evaluation, but that article has only 98 citations in 25 years. However, a similar empirical study by Hong et al. (1999), examining theories of intelligence using Dweck and Leggett's (1998) theoretical framework and their link with causal attributions of performance, has 1,957 citations. Importantly, adults and researchers may assume that growth mindset includes some level of nuance described in the less-cited research, but the majority of theoretical and empirical research using the fixed and growth mindset framework does not align with these assumptions. Instead, researchers using this framework define and measure intelligence mindset using more extreme or dichotomous terms and scale items that do not directly acknowledge the nuanced, middle-ground perspective held by some people that ability can change and that it can also stay the same.

Evidence of these dichotomous definitions and assessments of intelligence mindset are seen in the growth and fixed mindset items that are typically used in research. These items often include the word *always* or do not include specific language about the time or constancy of the belief in question. For example, some growth mindset items from one of the more popular intelligence mindset scales use the word *always* in their statement (e.g*., No matter how much intelligence you have, you can always change it quite a bit.*). Other items use vague terms for the amount of change in ability a person can accomplish (e.g., *You can substantially change how intelligent you are.*; Dweck et al., 1995; Henderson et al., 1992). Although children can understand nuances in ideas, children are less likely than adults to understand indirect or unstated nuances in a scale item unless it is explicitly spelled out for them (e.g., Ganley & McGraw, 2016). Thus, the items used in child-specific intelligence mindset scales should clearly and directly ask about the nuances that are important components underlying the construct.

5. The present study

The overall purpose of the present study is to refine the conceptualization of the construct of intelligence mindset within the mathematics domain. We aimed to understand how nuanced beliefs about mathematics intelligence malleability fit within the current growth and fixed mindset framework. We label these nuanced beliefs *mixed* intelligence mindsets in mathematics, which attempt to take into account intelligence beliefs that are somewhere in between being fully fixed and fully malleable. We have created a construct conceptualization, presented in Fig. 2, that serves to illustrate the previous conceptualizations of intelligence mindset (i.e., Dweck & Leggett, 1988) within the domain of mathematics as well as a new conceptualization of mathematics intelligence mindset that incorporates mixed mindset beliefs. Fig. 3 contains the proposed definitions of growth, fixed, and mixed mathematics intelligence mindset.

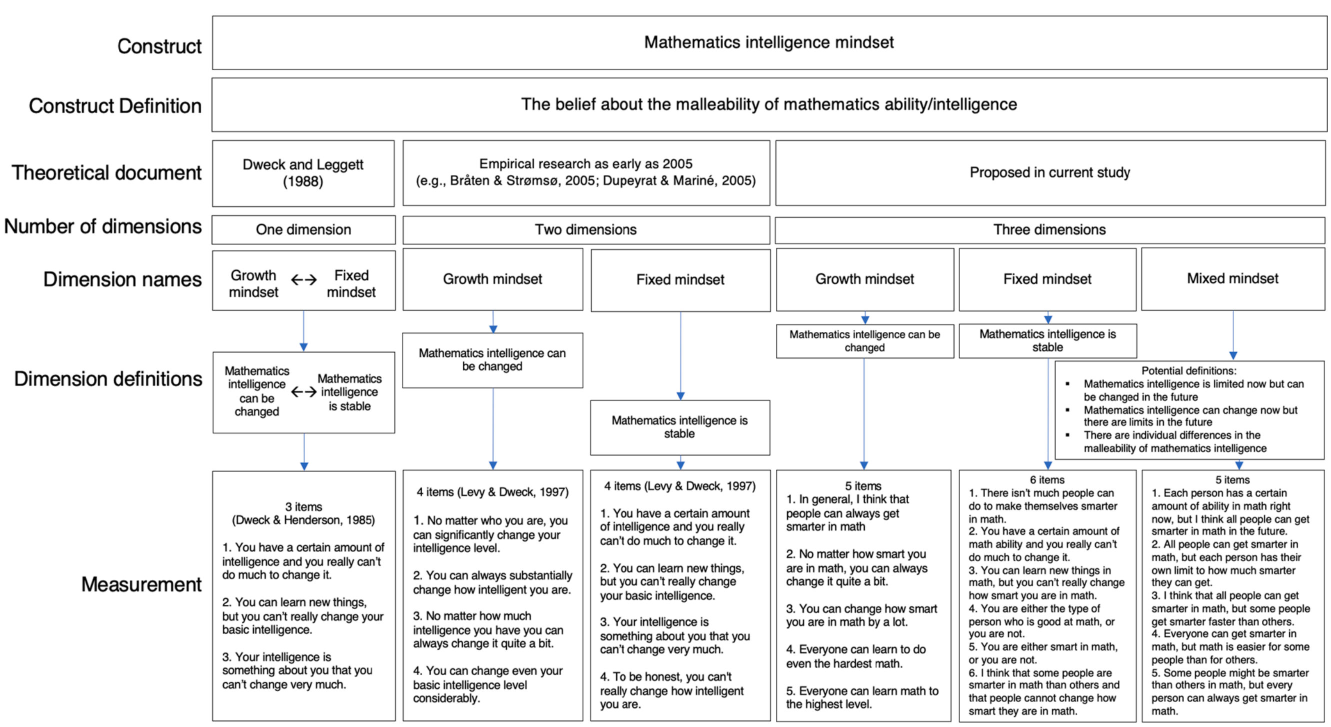

Fig. 2. Theorized construct conceptualization of mathematics intelligence mindset developed based on literature review.

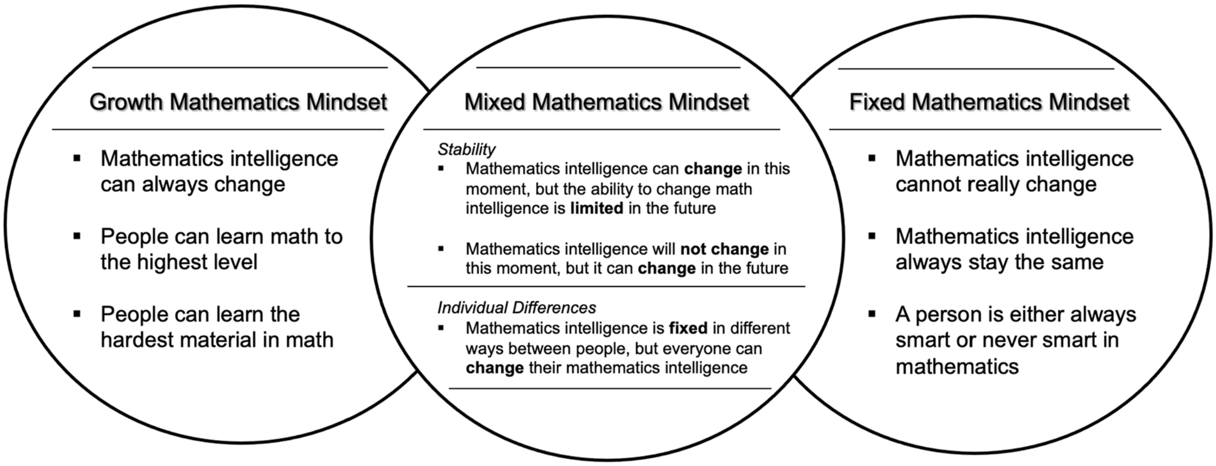

Fig. 3. General definitions of growth mindset, mixed mindset, and fixed mindset in mathematics.

For our study, we analyzed data from a large sample of third-grade students on a novel scale designed to assess their levels of growth, fixed, and mixed mathematics intelligence mindsets. We asked the following three research questions:

1. How related are mixed mathematics intelligence mindset items to growth and fixed mathematics intelligence mindset items?
2. How do different models of mathematics intelligence mindset fit when they include fixed and growth mindset items that map on to the definitions of the original growth and fixed mindsets and/or new mixed intelligence mindset items?
3. What is the relation between mathematics achievement and mathematics intelligence mindset when mindset is operationalized in different ways that include or do not include mixed intelligence mindset items?

For our first research question, we expected the mixed mindset items to have significant positive relations with both the growth mindset set of items and the fixed mindset set of items, given the intentional overlap of growth and fixed aspects in these new mixed mindset items. We expected that the mixed mindset items would have magnitudes of correlations with the growth and fixed mindset items similar to the correlations between the growth and fixed mindset items.

For our second question, we expected to find support for a multidimensional model of mathematics intelligence mindset with at least separate growth and fixed mindset factors (e.g., Bråten & Strømsø, 2005; Dupeyrat & Mariné, 2005; Kaijanaho & Tirronen, 2018). We hypothesized that models including mixed mindset items would have adequate fit to the data, but we did not have expectations about the relative fit of models with mixed mindset items compared to models without mixed mindset items. We also were unsure whether the mixed mindset items would fit better as a separate dimension or incorporated into models using the fixed and growth mindset framework (i.e., mixed mindset items cross-loaded onto growth and fixed factors).

For our third question, we expected to find a small, positive association ($r = .10$–$.30$; Cohen, 1988) between mathematics achievement and a latent growth mindset factor and a small, negative association between mathematics achievement and a latent fixed mindset factor (Sisk et al., 2018). We did not have a hypothesis for the size of a relation between latent factors that included the new mixed intelligence mindset items and mathematics achievement, but we expected at least a similar strength in relation to that of the growth mindset and fixed mindset relation with mathematics achievement.

6. Method

6.1. Participants

Data were collected from an initial sample of 785 third-grade students who had teachers participating in a larger, longitudinal project examining teacher's mathematics anxiety and instruction in relation to student mathematics achievement. Students were enrolled in public elementary school classrooms in 4 counties in the state of Florida in the United States. These schools ranged from 11% of students qualifying for free or reduced priced lunch to 94% qualifying, with a median of 62%. Sixty-six percent of schools were Title 1 schools. There were 87 students from the initial 785 student sample who did not take or answer any item from both the mathematics achievement test and the mathematics intelligence mindset scale. These students were removed from our data set, which left us with an analytic sample of 698 students (age data missing for 22.5% of sample; mean age for sample with age data = 9 years 3 months old, SD = 5.82 months, range = 7 years 6 months old − 11 years 10 months old). Available gender data on our sample indicated that 44% were girls, 47% were boys, and gender information was unavailable or not indicated for 9%. Race and ethnicity data were missing for almost 10% of the sample, 3% of students were Asian, 22% were Black, 11% were Hispanic, 11% were multiracial, and 42% were White. Additionally, 46% of students had free or reduced lunch status, 38% did not have this status, and 16% missing data for this status. See Part 1 of the Supplemental Materials for additional sample demographic details.

6.2. Measures

6.2.1. Mathematics intelligence mindset scale for children (MIMS-C)

We developed a new scale which we titled the Mathematics Intelligence Mindset Scale for Children (MIMS-C) to assess mathematics intelligence mindset. We adapted items from existing intelligence mindset measures (Dweck et al., 1995; Stipek & Gralinski, 1996) and developed new items with age-appropriate language that children could understand in the domain of mathematics. Students rated items using the following 5-point Likert scale: (1) I disagree, (2) I kind of disagree, (3) I don't disagree or agree, (4) I kind of agree, and (5) I agree. For all MIMS-C items, we first tested them in cognitive interviews with children to assess the interpretability of the items (see Part 2 of the Supplemental Materials for details). Items are available in Table 1.

Growth Intelligence Mindset in Mathematics. We used five items to measure growth intelligence mindset in mathematics. One item was adapted from a fixed mindset item used by Stipek and Gralinski (1996) and two were adapted from Dweck et al. (1995). We also developed two novel items.

Table 1
Descriptive statistics for MIMS-C items.

Mindset	Variable Name and Item	Adapted	N	% missing not PM	% missing and PM	Mean	SD	Skew	Kurt	all items item-total r^a	w/in subscale item-total r
Growth	GM1. I think that people can always get smarter in math	S&G	410	0.8	30.9	4.76	0.71	−3.77	15.33	.55	.56
	GM2. No matter how smart you are in math, you can always change it quite a bit.	Dweck	364	1.3	38.1	4.54	0.96	−2.36	5.09	.44	.52
	GM3. You can change how smart you are in math by a lot.	Dweck	600	0.2	0.0	4.46	0.97	−1.96	3.45	.40	.38
	GM4. Everyone can learn to do even the hardest math.	Novel	415	0.0	30.9	4.42	1.10	−1.95	2.82	.49	.49
	GM5. Everyone can learn math to the highest level.	Novel	368	0.7	38.1	4.26	1.24	−1.64	1.53	.40	.50
Fixed	FM6. I don't think there is much people can do to make themselves smarter in math.	S&G	366	1.0	38.1	1.86	1.34	1.36	0.47	-.45	.50
	FM7. You have a certain amount of math ability and you really can't do much to change it.	Dweck	407	1.3	30.9	2.07	1.42	0.98	−0.48	-.46	.55
	FM8. You can learn new things in math, but you can't really change how smart you are in math.	Dweck	411	0.7	30.9	1.94	1.43	1.21	−0.08	-.42	.47
	FM9. A person is either always a math person or never a math person.	Novel	410	0.8	30.9	2.53	1.43	0.40	−1.11	-.20	.33
	FM10. A person is either always smart in math or never smart in math.	Novel	597	0.7	0.0	2.26	1.38	0.64	−0.85	-.27	.35
	FM11. I think that some people are smarter in math than others and that people cannot change how smart they are in math.	Novel	371	0.2	38.1	1.83	1.34	1.37	0.45	-.41	.46
Mixed	MM12. Each person has a certain amount of ability in math right now, but I think all people can get smarter in math in the future.	Novel	414	0.2	30.9	4.50	0.97	−2.15	4.07	.38	.28
	MM13. All people can get smarter in math, but each person has their own limit to how much smarter they can get.[b]	Novel	601	0.0	0.0	3.36	1.67	−0.37	−1.54	-.10	.07
	MM14. I think that all people can get smarter in math, but some people get smarter faster than others.	Novel	413	0.3	30.9	4.13	1.25	−1.35	0.71	0.24	0.23
	MM15. Everyone can get smarter in math, but math is easier for some people than for others.	Novel	409	1.0	30.9	4.43	1.05	−1.99	3.22	0.28	0.24
	MM16. Some people might be smarter than others in math, but every person can always get smarter in math.	Novel	599	0.3	0.0	4.76	0.70	−3.58	13.70	0.46	0.28

Notes. [a]fixed mindset items were reverse-coded for all item-total *rs*; [b]item was dropped from the scale due to low item-total correlations and correlations with other mixed mindset items; Item-total *r* indicates correlation between single item and the remaining items; All items had a minimum score of 1 and maximum score of 5; S&G = Stipek & Gralinski (1996); Dweck = Dweck, Chiu, & Hong (1995); Novel = newly-developed item; PM = planned missing; Skew = skewness; Kurt = kurtosis.

Fixed Intelligence Mindset in Mathematics. We used six items to measure fixed intelligence mindset in mathematics. One item was adapted from Stipek and Gralinski (1996) and two were adapted from Dweck et al., (1995). We also developed three novel items.

Mixed Intelligence Mindset in Mathematics. We developed an initial set of five mixed mindset items using the two definitions presented in the introduction about potential ways to characterize a mixed intelligence mindset in mathematics. One item asked about the belief that a person has a certain amount of mathematics intelligence in the present moment (fixed), but that mathematics intelligence can be increased in the future (growth). Another item asked about the belief that a person's intelligence can be *changed and increased* (growth), but there is a capacity or limit that it will eventually reach *in the future* (fixed). Three items asked about the belief that there are individual differences in mathematics ability, and these differences could manifest themselves in people's starting levels of ability, growing levels of ability, or both.

6.2.2. *Mathematics assessment*

Students completed the 28-item Elementary Mathematics Student Assessment (EMSA; Schoen et al., 2021) for third-grade students, which aligns with the expectations in the state curriculum standards and designed for students to take at the end of third grade. Using items that measure competence in word problems, number relations, fractions, basic number facts, and computation, the EMSA measures knowledge in the domains of number, operations, and algebraic reasoning. Dimensionality analysis suggested that the EMSA test measures a single, dominant factor. The EMSA score is based on item-response theory and uses a two-parameter logistic model to create a theta (ability) estimate for each student, with a higher score indicating a higher level of mathematics achievement. Marginal reliability for the third-grade test was 0.88 (Schoen et al., 2021).

6.3. *Procedure*

The data analyzed in the current paper were collected during spring of the 2019 academic school year (i.e., second half of third grade). We employed planned missing data collection methods for the MIMS-C to reduce survey assessment time (Little & Rhemtulla, 2013). Teachers who were participating in the longitudinal project were randomly assigned to one of three sets of student test packets and asked to administer these packets to the students in their classrooms. These packets contained the battery of tests and surveys administered for the project, including the entire mathematics achievement test as well as a subset of the MIMS-C items based on the planned missing design (see *Analysis Plan* for details). Teachers were advised to have their class complete the mathematics test and the battery of surveys and questionnaires on separate days and only a few days apart. Prior to the students completing the survey, teachers read through some practice items with students to have them practice responding to Likert scale items. Students then read the mindset items on their own. The mathematics assessment items were read aloud by the teacher. The packets also included additional measures that were beyond the scope of the present study.

Teachers were paid for their time spent on research activities that occurred outside of their school day (i.e., measures beyond the scope of the current study), and students were rewarded with a pencil for their participation, in accordance with the incentivization protocol #2019.28632 approved by the institutional review board at the university.

6.4. *Analysis Plan*

Our analyses consisted of inter-item correlations among the MIMS-C items, comparisons of models with and without the new mixed mindset items through the use of confirmatory factor analyses (CFA) to test for best fit of competing models, and correlations among latent mindset factors and mathematics achievement. We estimated correlations and conducted CFA in Mplus version 8.6 using the TYPE = COMPLEX command to adjust the standard errors based on the grouping variable of teacher (Muthén, 1997; Muthén & Muthén, 2016; Stapleton et al., 2016). For any correlational analyses, we expected that even small correlation coefficients (i.e., $r = .10$) would be statistically significant.

Our methods and a majority of our analyses were adapted from our online preregistration of this study available on Open Science Framework (https://osf.io/2tk5c/). Some analyses and details were added beyond those listed in the preregistration. The differences between the analyses in the preregistration and analyses in this paper are listed in Part 3 of the Supplemental Materials. An exploratory analysis (i.e., correlation magnitude comparisons) was also added for the third research question to provide further information about the difference in magnitude of correlations.

6.4.1. Missing data

Due to the planned missing data collection, students received 75% of the items in the mathematics intelligence mindset scale. The items they received were based on their teacher's random assignment to one of three student packets. We used the WLSMV estimator to account for the categorical nature of the item responses, estimate parameters, and to provide us with the ability to compare nested models. The sub-sample for analyses with only the MIMS-C items was made up of data from 601 students out of the 698 in the analytic sample, because only 601 students had at least one mindset item completed, and the WLSMV estimator uses pairwise deletion (Asparouhov & Muthen, 2010).

6.4.2. CFA indices

We used the following model fit indices (commonly used for continuous variables) to evaluate and compare each of our models: chi-square ($\chi 2$) *p*-values greater than 0.05, root mean square error of approximation (RMSEA) values less than 0.08, and Tucker-Lewis index (TLI) and comparative fit index (CFI) values greater than or equal to 0.90 (Hu & Bentler, 1999; Kline, 2015). We made modifications based on factor loading coefficients reported in Mplus that were conceptually reasonable (e.g., non-significant or fair-to-poor factor loadings). We considered standardized factor loadings greater than 0.72 to be excellent, between 0.63 and 0.71 to be very good, between 0.55 and 0.62 to be good, between 0.33 and 0.45 to be fair, and less than 0.32 to be poor (Comrey & Lee, 1992).

6.4.3. Reliability indices

We created 10 imputed data sets in SPSS version 28 to fill in missing MIMS-C data from the 601 students who had at least one intelligence mindset item. We then calculated coefficients omega from each data set and pooled them together to obtain an overall average reliability coefficient of omega (Green & Yang, 2009).

7. Results

7.1. Descriptive statistics

Descriptive statistics for the MIMS-C items are available in Table 1. On the non-missing data, the mean scores for the growth mindset items were high on the 1–5 rating scale ($M = 4.26$ to 4.76). Mean scores for the fixed mindset items were low ($M = 1.83$ to 2.26). The mean for four out of five of the mixed mindset items were high ($M = 4.13$ to 4.76). Twelve of the 16 MIMS-C items' skewness values were between –2 and 2 (Tabachnick & Fidell, 2013). Most of the items were in normal range for kurtosis with values between –7 and 7 (Hair et al., 2010), although many were close to the cutoff values. One growth mindset and one mixed

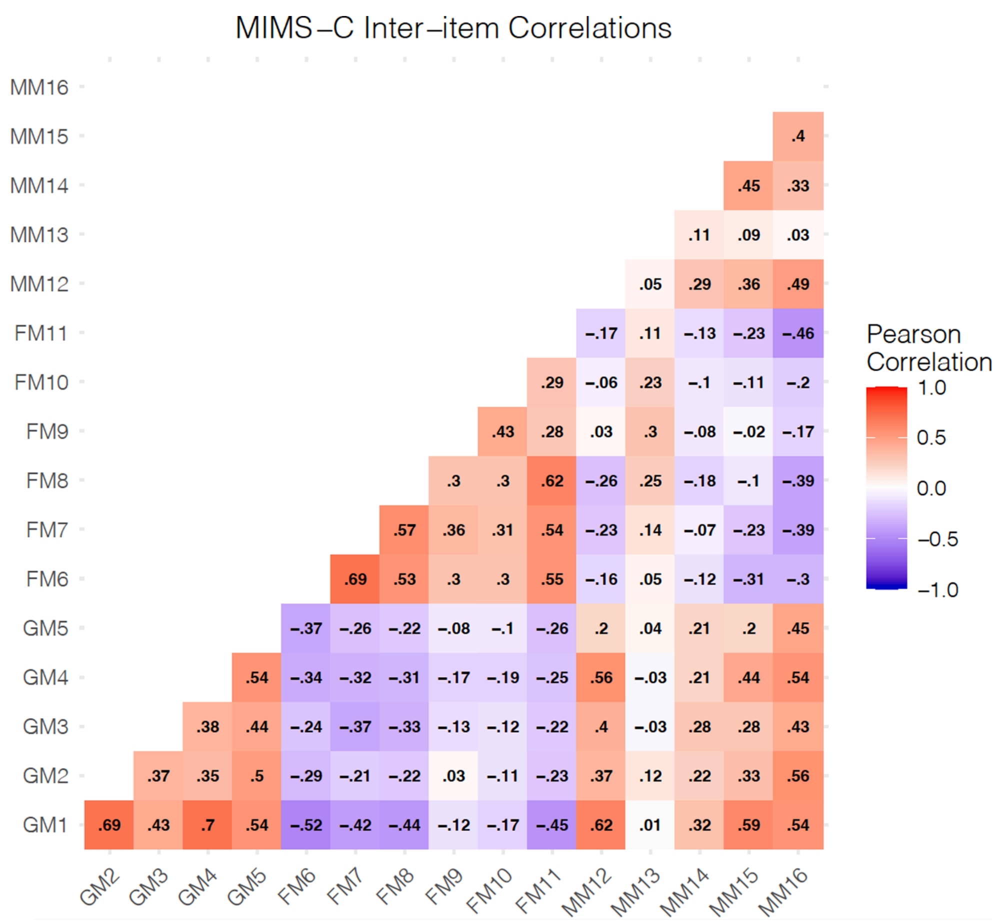

Fig. 4. Heatmap of inter-item correlations between the MIMS-C items ($N = 601$). Correlations between −.07 and .07 were not significant at $p < .05$.

mindset item did have kurtosis values beyond these cutoffs (i.e., 15.33 and 13.70, respectively).

7.2. Research question 1: Relations between mixed, growth, and fixed mathematics intelligence mindset items

To examine the relations between the items in the MIMS-C, we examined inter-item correlations, which are available in the heat map in Fig. 4. Correlations below .07 (absolute value) were not statistically significant. Patterns were evident between the items within each dimension. Growth mindset items had moderate-to-strong, positive correlations with other growth mindset items (*rs* = .35–.70). Fixed mindset items had weak-to-strong, positive correlations with each other (*rs* = .28–.69). Most mixed mindset items had weak-to-moderate correlations with each other (*rs* = .29–.49). There was one notable exception, specifically item 13 (*All people can get smarter in math, but each person has their own limit to how much smarter they can get*). The correlation coefficients with item 13 never exceeded .30 for any other item and were near zero for the rest of the mixed mindset items (*rs* = .03–.11). Due to these low correlations, we dropped item 13 from further analyses.

All the growth mindset items had negative correlations (between -.52 and -.20) with three fixed mindset items and weak-to-no relations with the other two fixed mindset items (*rs* between -.19 and .003). The growth mindset items had small-to-large, positive correlations with the four mixed mindset items (*rs* between .20 and .62). Three of the fixed mindset items (items FM6, FM8, and FM11) had small-to-moderate negative correlations with the mixed mindset items (*rs* between -.46 and -.12). One fixed mindset item (FM7) also had small-to-moderate negative correlations with the mixed mindset items with the exception of one pair that had no significant relation (FM7 and MM14; *r* = -.07). The remaining two fixed mindset items (FM9 and FM10) had weak-to-no correlation with the mixed mindset items (*rs* between -.20 and .03).

7.3. Research question 2: Testing models of mathematics intelligence mindset

7.3.1. CFA and reliability

To gauge the fit of different mathematics intelligence mindset models, we obtained model fit indices through CFA from two fixed and growth mindset models and three mixed mindset models. Table 2 shows model fit indices and Figs. 5 and 6 present the standardized factor loadings for items in each model. We also tested supplemental post-hoc models with the mixed mindset items and either the growth or fixed mindset items (See Part 4 of the Supplemental Materials).

Table 2
Goodness-of-fit indicators from confirmatory factor analyses of mathematics intelligence mindset models.

Model	χ^2	*df*	*p*	CFI	TLI	RMSEA (90% CI)
Fixed and Growth Mathematics Mindset Models						
One-factor model with combined growth and fixed mindset factor	172.81	44	<0.001	0.899	0.873	0.070(0.059–0.081)
Two-factor model with separate growth and fixed mindset factors	66.97	43	0.011	0.981	0.976	0.030(0.015–0.044)
Mixed Mathematics Mindset Models						
Two-factor model with separate growth and fixed mindset factors including cross-loaded mixed mindset items	116.06	85	0.014	0.981	0.976	0.025(0.012–0.035)
Modified two-factor model with combined growth and mixed mindset factor and separate fixed mindset factor	124.46	89	0.008	0.978	0.974	0.026(0.014–0.036)
Three-factor model with separate growth, fixed, and mixed mindset factors	121.91	87	0.008	0.979	0.974	0.026(0.014–0.036)

Notes. *N* = 601; Estimator used was Weighted Least Squares Means Variances.

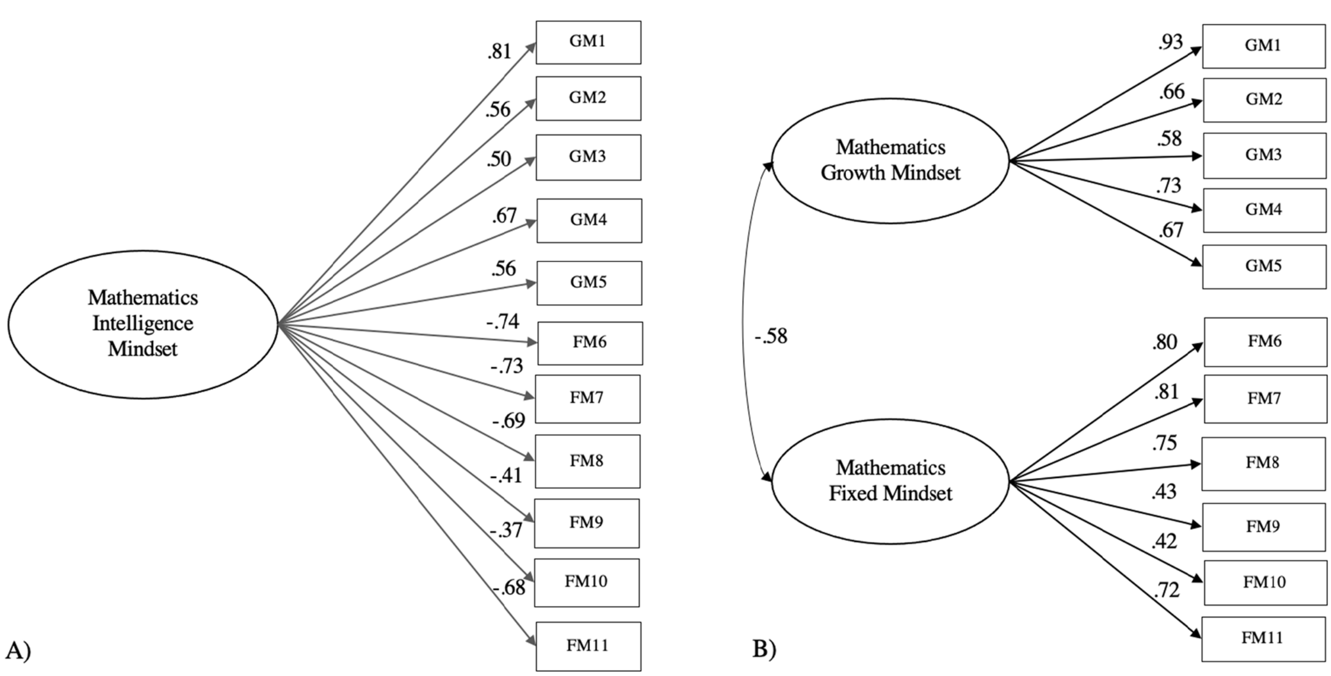

Fig. 5. Standardized factor loadings for two traditional mathematics intelligence mindset models with growth and fixed mindset items. Model A) shows a unidimensional model and Model B) shows a two-factor model. All factor correlations and standardized factor loadings are significant at $p < .0001$.

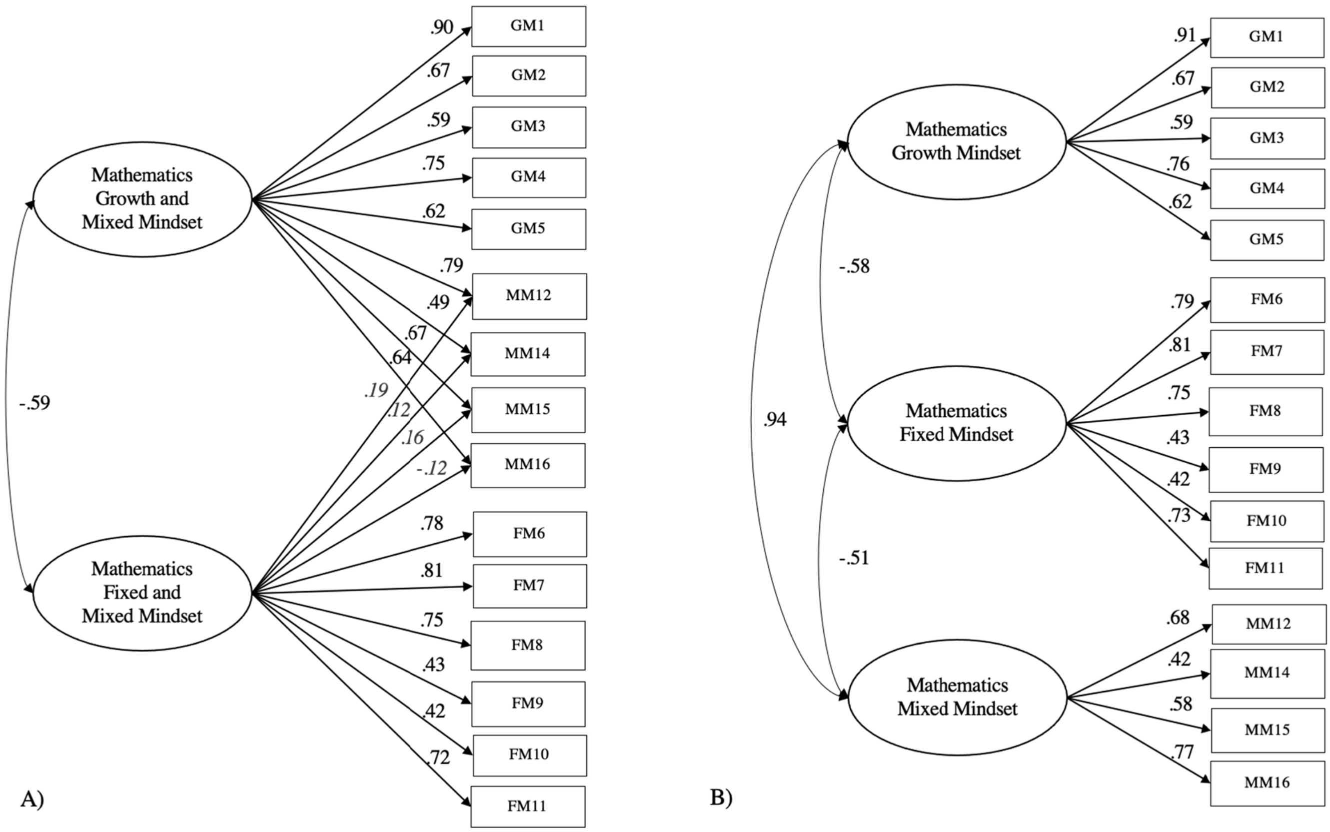

Fig. 6. Standardized factor loadings for two newly conceptualized mathematics intelligence mindset models with growth, fixed, and mixed mindset items. Model A) shows a two-factor model and Model B) shows a three-factor model. Italicized factor loadings are not significant. All factor correlations and other standardized factor loadings are significant at $p < .0001$.

Fixed and Growth Mathematics Mindset Models. We first tested a one-factor model with just growth and fixed items. This model had an acceptable RMSEA value, a significant chi-square test, and CFI and TLI values slightly below the threshold for the acceptable range ($\chi2_{(44)} = 172.81$, $p < 0.001$, RMSEA = 0.070 [90% CI = 0.059 – 0.081], CFI = 0.899, and TLI = 0.873). Factor loadings were all significant and had an absolute value greater than 0.36. The mean ordinal omega coefficient for this unidimensional mindset scale was 0.84.

We then tested a two-factor correlated traits model with only growth and fixed mindset items. This model had good fit, with the exception of the chi-square test ($\chi2_{(85)} = 66.97$, $p = .011$, RMSEA = 0.030 [90% CI = 0.015 – 0.044], CFI = 0.981, and TLI = 0.976). The growth mindset factor and fixed mindset factor had a significant negative correlation of -.58. The factor loadings were all significant, and factor loadings were greater than 0.43. The ordinal omega coefficient was 0.73 for both the growth mindset and the fixed mindset subscales, separately.

Mixed Mathematics Mindset Models. We next tested a two-factor correlated traits model with the mixed mindset items cross-loaded onto both the growth and fixed factors. This model had good fit indices ($\chi2_{(85)} = 116.06$, $p = .014$, RMSEA = 0.025 [90% CI = 0.012 – 0.035], CFI = 0.981, and TLI = 0.976). The growth mindset factor and the fixed mindset factor had a significant negative correlation of –.59. All the factor loadings onto the growth mindset factor for the growth mindset and mixed mindset items were significant and greater than 0.49. The factor loadings for the fixed mindset items loading onto the fixed mindset factor were all significant and greater than 0.42. However, the factor loadings for the four cross-loaded mixed mindset items on the fixed mindset factor were not statistically significant, and the loadings were small in magnitude (less than 0.19).

We modified this two-factor model by removing the four non-significant cross-loaded mixed mindset items from the fixed mindset factor (Fig. 7). These modifications left a model that is characterized by a combined growth and mixed mindset factor and a separate fixed mindset factor ($\chi2_{(89)} = 124.46$, $p = .008$, RMSEA = 0.026 [90% CI = 0.014 – 0.036], CFI = 0.978, and TLI = 0.974). All factor loadings were significant and greater than 0.40. The combined growth and mixed mindset factor and the fixed mindset factor had a significant negative correlation of –.56, again indicating that an individual can have varying levels of a combined growth and mixed mindset and a fixed mindset. The mean ordinal omega coefficient was 0.73 for the fixed mindset subscale and 0.82 for the combined growth and mixed mindset subscale.

We also tested a three-factor correlated traits model with separate growth, fixed, and mixed mindset factors. This model also had good fit indices, ($\chi2_{(87)} = 121.91$, $p = .008$, RMSEA = 0.026 [90% CI = 0.014 – 0.036], CFI = 0.979, and TLI = 0.974). The factor loadings for the indicators on each of the three factors were all statistically significant ($p < .001$) and greater than 0.42. The correlation between the growth mindset factor and the fixed mindset factor was -.58 and between the fixed mindset factor and mixed mindset factor was -.51. The correlation between the growth mindset factor and the mixed mindset factor was very high at .94. The mean ordinal omega coefficients were acceptable for both the growth mindset subscale ($\omega = 0.73$) and the fixed mindset subscale ($\omega = 0.73$). However, for the mixed mindset subscale, the mean ordinal omega coefficient was poor at 0.54.

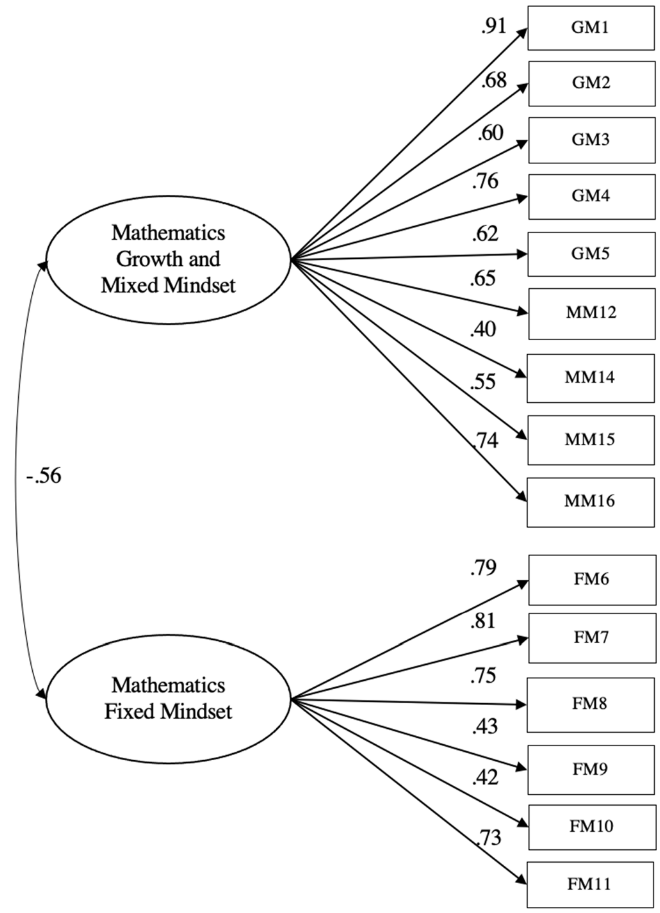

Fig. 7. Standardized factor loadings for the modified best-fitting two-factor model of mathematics intelligence mindset with growth, fixed, and mixed mindset items. The factor correlation and all standardized factor loadings are significant at $p < .0001$.

7.3.2. Comparison of nested and non-nested models

We compared the fit of nested models using the DIFFTEST option in Mplus (Asparouhov et al., 2006). The DIFFTEST results comparing the fixed and growth unidimensional and two-dimensional mindset models was statistically significant ($\chi2_{(1)} = 64.70, p < .0001$). This indicated that the better fitting model was the two-factor model. The DIFFTEST results comparing the two-factor mindset model that included a combined mixed and growth mindset factor against the three-factor mixed mindset model was not statistically significant ($p = .19$). This result suggested that the modified two-factor mixed mindset model has similar fit to the three-factor model.

We used model fit and reliability indices to compare the non-nested fixed and growth mindset two-factor model and the modified two-factor mixed mindset model with a growth/mixed factor and a fixed factor. The fixed and growth two-factor model was more parsimonious as it had fewer items in the model and therefore had fewer parameters to calculate than the two-factor mixed mindset model. The reliability index for the items in the combined growth mindset and mixed mindset factor ($\omega = 0.82$) in the mixed mindset two-factor model was better than the reliability index for the items in the growth mindset factor for the fixed and growth mindset model ($\omega = 0.73$).

7.4. Research question 3: Relations between mathematics achievement and mathematics intelligence mindsets

We obtained correlation coefficients to test the relation between mathematics achievement and different conceptualizations of mathematics intelligence mindset as measured through latent factors from the various models compared in Research Question 2. Table 3 presents these correlations. All correlation coefficients were statistically significant, though they varied in magnitude.

The factor score from the unidimensional fixed and growth mindset model, where a higher score indicated greater agreement with a growth mindset and a lower score indicated greater agreement with a fixed mindset, had a moderate, positive correlation with mathematics achievement ($r = .31, p < .0001$).

In the two-factor fixed and growth mindset model, the relation between the growth mindset factor and mathematics achievement was small and positive ($r = .19, p < .0001$). The relation between the fixed mindset factor and mathematics achievement was moderate and negative ($r = -.34, p < .0001$). We were further interested in testing whether there were significant differences in the strength of the correlations for the fixed mindset factor and the growth mindset factor with mathematics achievement. In an exploratory analysis comparing the correlation coefficients (i.e., not preregistered), we found that the correlation between fixed mindset and mathematics achievement was statistically stronger than that for growth mindset and mathematics achievement ($z = 4.47, p < .0001$).

When modeled in the two-factor mindset model with a fixed mindset factor and separate factor with combined growth and mixed mindset items, the combined growth and mixed mindset factor had a moderate positive correlation with mathematics achievement ($r = .25, p < .0001$). Fixed mindset, modeled in the same way as the two-factor fixed and growth mindset model, had a moderate negative correlation with mathematics achievement ($r = -.34, p < .0001$). In an exploratory correlation comparison analysis, we found that the correlation between

Table 3
Correlations between latent factors from mathematics intelligence mindset models and mathematics achievement.

Model	Combined fixed and growth factor	Fixed factor	Growth factor	Combined growth and mixed factor	Mixed factor
Fixed and Growth Mathematics Mindset Models					
One-factor model with combined growth and fixed mindset factor	.31**	–	–	–	–
Two-factor model with separate growth and fixed mindset factors	–	-.19**	.34**	–	–
Mixed Mathematics Mindset Models					
Modified two-factor model with combined growth and mixed mindset factor and separate fixed mindset factor	–	-.34**	–	.25**	–
Three-factor model with separate growth, fixed, and mixed mindset factors	–	-.34**	.19**	–	.34**

Notes. N = 601; *$p < .001$; **$p < .0001$; Lower scores for the one-factor model indicates greater agreement with fixed mindset and higher scores indicate greater agreement with growth mindset.

fixed mindset and mathematics achievement was statistically stronger than that for the combined growth and mixed mindset and mathematics achievement ($z = 2.68$, $p = .004$).

In the three-factor model, growth mindset alone had a small positive correlation with mathematics achievement ($r = .19$, $p < .0001$). Mixed mindset alone had a moderate positive correlation with mathematics achievement ($r = .34$, $p < .0001$). Fixed mindset again had the same moderate negative correlation with mathematics achievement ($r = -.34$, $p < .0001$). Our exploratory comparison analysis indicated that the correlation between mixed mindset and mathematics achievement was statistically stronger than that for growth mindset and mathematics achievement ($z = 11.88$, $p < .0001$). The correlation between fixed mindset and mathematics achievement was also statistically stronger than that for growth mindset and mathematics achievement ($z = 4.52$, $p < .0001$). The correlations between fixed mindset and mathematics achievement and between mixed mindset and mathematics achievement were similar in magnitude ($z = 0.00$, $p = .50$).

8. Discussion

Across studies, researchers typically use the growth and fixed mindset framework to conceptualize intelligence mindsets. However, theoretical and empirical evidence suggests that beliefs about intelligence are not strictly oriented towards growth or fixed mindsets (e.g., Boyum, 1988; Droege, & Stipek, 1993). In the present study, we explored the presence of nuanced beliefs about the malleability of mathematics intelligence in a sample of third-grade students. We also examined how similar these nuanced beliefs are to those beliefs defined by the growth and fixed mindset framework and their relation with mathematics achievement.

8.1. Children's beliefs about the malleability of mathematics intelligence

In keeping with previous research, we found that students tended to largely agree with growth mindset items assessing the belief that mathematics intelligence is changeable and disagree with fixed mindset items assessing the belief that intelligence cannot be changed (Dweck et al., 1995; Dweck & Henderson, 1989). We also found that students tended to agree with the new mixed mathematics intelligence mindsets that we developed. These findings provide preliminary evidence that children do hold nuanced beliefs about the malleability of mathematics intelligence that are between growth and fixed beliefs, as we have defined them. Some early studies excluded participants who had beliefs that did not clearly fit into either a growth or fixed intelligence mindset category (e.g., Dweck & Henderson, 1989). However, our study results challenge the practice of removing in-between scores from analyses and present the possible addition of measuring these nuanced beliefs about mathematics ability in children.

Further, the responses to a few growth and mixed mindset items were positively skewed. Previous research has suggested social desirability bias can lead to skewness of responses on growth mindset items on college students (Dweck & Leggett, 1988). Other work with secondary students has examined and not found a relation between socially desirability and responses to fixed-oriented intelligence mindset items (King, 2020). It remains unclear whether social desirability played a role in the skewness we found with the growth and mixed mindset items in our measure of mathematics intelligence mindset.

8.2. Relations between mixed, growth, and fixed mathematics intelligence mindset items

We found that growth, fixed, and mixed mindset items were mostly related to items in their own item set. The growth and fixed mindset items had mostly moderate-to-large correlations with items in their own item set. Most of the mixed mindset items also had moderate associations with each other, with the exception of one mixed mindset item that was designed to ask about beliefs that current mathematics ability can grow but future ability to change mathematics ability is limited. Interestingly, the strongest correlation this misfit item had with other items were small but mostly positive correlations with several fixed mindset items. It may be that this mixed mindset item had a connection with the fixed mindset item based on the fixed aspect of the mixed mindset statement (i.e., *each person has their own limit to how much smarter they can get*).

We expected to find relations of similar magnitudes between items across different item sets, particularly positive associations between mixed mindset and growth mindset items as well as between mixed mindset and fixed mindset items. We did find positive correlations, ranging from small to large, between the four mixed mindset items and the growth mindset items that supported our hypothesis. In contrast to our initial hypotheses though, we found negative correlations ranging from small to medium between the mixed mindset items and the fixed mindset items. This suggests that the overlapping fixed language in the mixed mindset items was not distinct enough to warrant similar ratings in a positive direction to those of the fixed mindset items. We did find the growth and fixed mindset items had mostly small-to-medium negative relations with each other, which overall suggests that the mixed mindset items have correlations of similar magnitude to fixed and growth mindset items to those correlations between growth and fixed mindset items. The existence of larger-than-zero correlations between two items does not necessarily imply measurement of the same construct, but the following model comparisons provide a better look into the fit of the mixed mindset dimension in the construct of mathematics intelligence mindset.

8.3. Considering mixed mindset in models of mathematics intelligence mindset

To understand how the new mixed mindset items assessing nuanced beliefs fit within the mindset framework, we tested the performance of models that included only growth and fixed intelligence mindset items about mathematics as well as models that included mixed mathematics intelligence mindset items. We found support for a multidimensional model, as we had expected. This finding is not consistent with Dweck and Leggett's (1988) original unidimensional model but does align with recent factor analytic evidence suggesting that growth and fixed mindsets in mathematics should be assessed separately (Dupeyrat & Mariné, 2005; Park et al., 2016; Shih, 2011).

Although we were unsure of how a model with mixed mindset items would compare to models without these new items, we had hoped that our results would help us determine a superior model to use to conceptualize the construct of mathematics intelligence mindset in third grade students. However, our dimensionality analyses and model comparisons did not provide clear results. Nevertheless, we were able to gain some insight into the usefulness of the mixed mindset items in the models.

We found adequate fit for a two-dimensional correlated trait growth and fixed mindset model, a two-dimensional correlated trait mixed mindset model that consisted of a combined growth and mixed mindset factor and a separate fixed mindset factor, and a three-dimensional correlated trait mixed mindset model that consisted of separate growth, mixed, and fixed mindset factors. The more complex models did not have significantly poorer fit to the data than the more parsimonious models. In other words, the nested mixed mindset models were not statistically different from each other, and the CFA indicators were also not clearly worse for the two-factor fixed and growth mindset only model than for any of the mixed mindset models. The parsimony principle is often applied to situations in which a group of models fit similarly to each other (Meehl, 1990). While this principle does not remove more complex models from the running to measure a construct, it does suggest the selection of the simplest model as the best model to use in analyses assessing the construct in question. Based on the parsimony principle alone, the best model to select in our case would be the two-factor fixed and growth mindset model due to the fewer parameters in this model. However, it is important to use the parsimony principle alongside theory to dictate model selection (Preacher, 2006).

Our theoretical, conceptualized model of mathematics intelligence mindset dictated that mathematics intelligence mindset could include mixed mindset in its framework. In addition to the similar fit across the different multidimensional models, the reliability index for the mixed mindset and growth mindset items together (0.82) was better than the index for the growth mindset items on their own (0.73). It is possible that this higher omega value is due to the greater number of items in the combined factor than in the separate factors (Ercan et al., 2007). Yet, we also found a strong correlation between the separate growth and mixed mindset factors. In totality, these results suggest that the growth and new mixed mindset items might be measuring the same latent trait of mathematics intelligence mindset.

8.4. *Relations between mathematics achievement and mathematics intelligence mindsets*

Based on the findings from Sisk et al. (2018), we hypothesized small and similar correlations between mathematics achievement and each mathematics intelligence factor within each model. Our hypotheses were mostly supported in our findings. When operationalized as a unidimensional fixed and growth mindset factor, where higher scores reflected higher growth mindset and lower scores reflected higher fixed mindset, we found that mindset had a moderate correlation with mathematics achievement ($r = .31$). Sisk et al. (2018) interpreted their intelligence mindset variable in a similar fashion to this mindset model, but they found a smaller sized correlation overall with achievement ($r = .10$) than we did in this study. The moderate correlation we found may be due to the mathematics-specific nature of these variables in our study or the specific age of our child sample. Sisk et al. (2018) also included measures that operationalized intelligence mindset in various ways (i.e., unidimensional, multidimensional) with a combination of growth and/or fixed mindset items in their scales, which might differ from the model we tested with only growth and fixed mindset items. These varying methods and sample characteristics may contribute to the differences in the strength of the correlation between our study and previous ones.

We ended up finding small correlations between growth mathematics mindset and mathematics achievement, but moderate correlations between fixed mathematics mindset and mathematics achievement and between the factors that included mixed mindset items and mathematics achievement. These correlations, and the significant increase in the correlation size between mathematics achievement and the growth mindset factor after including the mixed mindset items, suggest important links between the mixed mindset items and mathematics achievement. The relation between growth mindset and mathematics achievement is strengthened when growth mindset items allow for more nuance in their measurement instead of the rigid and constant definition of growth that is typically assessed in growth mindset items.

8.5. *Implications of incorporating mixed mindset in mindset framework*

What do our study findings contribute to what is known about the construct of mathematics intelligence mindset? As a whole, our findings present initial support for the potential expansion of at least the growth mindset definition within the current fixed and growth mindset framework. Our study provides evidence of the presence of nuanced beliefs about the malleability of mathematics intelligence in children that are clearly important to recognize and measure. In its current definition, a growth mindset in mathematics is the belief that mathematics intelligence is malleable and can continually change (i.e., Dweck & Leggett, 1988; Dweck & Yeager, 2019). We found in our study that agreement with growth mindset items does not necessarily preclude the possibility of fixed components in mathematics intelligence. Thus, we propose that

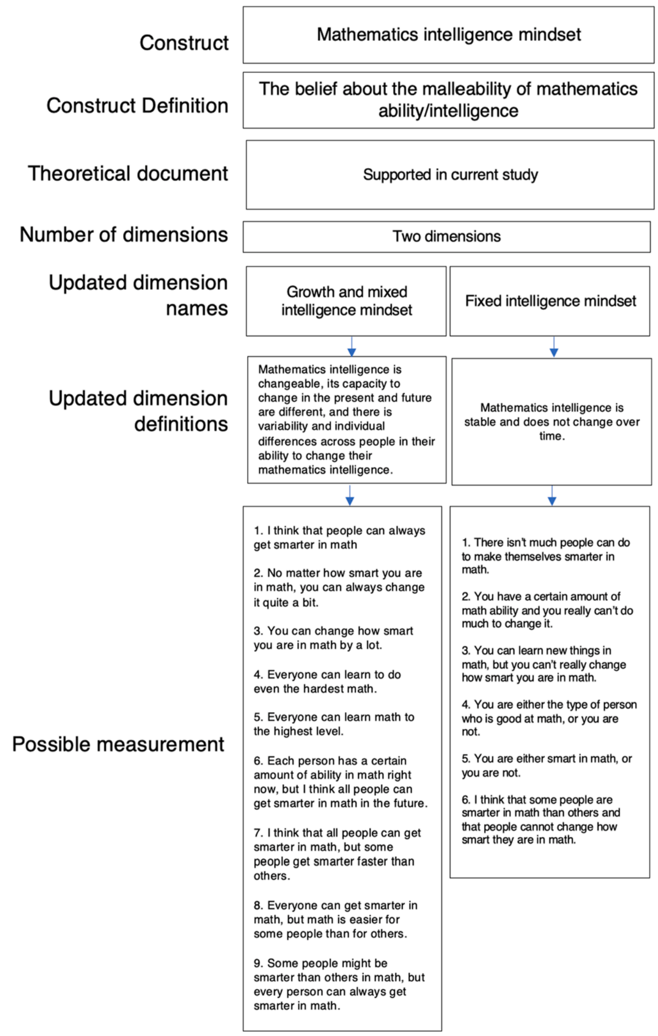

Fig. 8. Updated construct conceptualization of mathematics intelligence mindset based on evidence from the present study.

it is important for researchers in the intelligence mindset field concerned with the domain of mathematics to consider an expansion of the definition of growth mathematics intelligence mindset in children to include less extreme, mixed beliefs about the malleability of mathematics intelligence where there is recognition of fixed aspects in this trait. These changes can be done through the additional assessment of mixed components when assessing growth mindsets in mathematics in children. We present an updated multidimensional conceptualization of the construct of mathematics intelligence mindset that includes a fixed mindset dimension and a separate dimension made up of both mixed mindset and growth mindset (Fig. 8).

Our findings also suggest the need to measure fixed mindset in mathematics separately. Fixed and growth beliefs are not unidimensional in the domain of mathematics and should not be thought of as beliefs on a spectrum. Our supplemental analyses further suggest that the fixed and mixed mindset items together are not unidimensional. The continual finding of the fixed mindset items having better model fit when they are isolated in their own factor indicates that fixed mindset is its own distinct dimension and is discernible from both a growth mindset and a mixed mindset. The correlations between the fixed mindset items and the removed mixed mindset item (asking about growth in the current moment but limitations in future mathematical ability) suggest the possibility of some types of mixed mindset items contributing to the conceptualization and definition of fixed mindset. However, more research is needed to better understand whether there are fixed-leaning mixed mindset items that can expand the rigid conceptualization of fixed mindset that predominates the research literature.

8.6. Implications of mixed mindset in mathematics for educators and parents

Our study has implications for several areas of practice. The moderate, positive link between mathematics achievement and mixed mindsets that is stronger than the link between mathematics achievement and growth mindsets puts forward the possibility that mixed mindsets might also have stronger associations with other mathematics achievement behaviors and motivations. For example, students who hold mixed mindsets may experience less frustration due to the flexibility that this mindset can provide to students in challenging times. Recent research has shown that growth mindset messages can frustrate students when their efforts are applied in vain (Daniels et al., 2022). If a student holds a flexibility or situation-influenced mindset that characterizes a mixed mindset, it might enable them to remain motivated to learn and persist in the face of challenges as compared with those who hold more rigid fixed or growth mindsets. Despite this potential, more research is needed to investigate these speculated rewards of students endorsing mixed mindset about the malleability of their mathematics intelligence.

Our study results can inform the types of messages that practitioners, educators, and parents use to promote adaptive achievement behaviors in the mathematics classroom or home. The current messaging typically includes language most often used in growth mindset items (e.g., “You can always grow your intelligence”). Sometimes messages attribute the change in ability to specific factors like application of effort and hard work (e.g., “You must have worked hard on these problems” from Mueller & Dweck [1998]; “the brain could grow and get stronger through practice” from Blackwell et al. (2007)). Amemiya and Wang (2018) found that messages focused on effort alone can be interpreted as a sign of low innate ability by adolescents. As previously mentioned, some students who are told that intelligence can always change can become frustrated when their efforts are not met with success (Daniels et al., 2022). Thus, growth mindset messages may contradict students’ real experiences or discredit their previous learning or present ability levels.

Nuanced mindset messages, on the other hand, can acknowledge challenges and feel more valid for students in situations where they need more motivation to learn. For example, messages can include information about individual differences between people in the ability to change intelligence, such as “It can be easier for some people than for others, but everyone can get smarter in math.” or “I know it seems like other kids are getting this already, and that’s ok! Everyone learns at their own pace!” Messages might also highlight differences in the stability of intelligence or across time, such as “It’s ok! I know this feels so hard to figure out right now, but we’re going to keep working on it and you will get there!” To test for the potential benefits of nuanced mindset messages, future research should develop interventions with mixed mindset messages that parents and teachers can apply in mathematics contexts to investigate their impact on children’s achievement and motivation.

8.7. Limitations

There are several limitations in the current study that should be noted. First, although children in third grade—or around 8 to 9 years old—are often able to take more nuanced information into consideration (Eccles, 1999), some of the MIMS-C items may require children to think about their agreement with two potentially different pieces of information simultaneously (i.e., asking children to agree with the entire item that consisted of the two following parts: *All people can get smarter in mathematics* and *but each person has their own limit to how much smarter they can get.*). The cognitive interviews we conducted with children to assess the extent to which these items were understood and appropriately interpreted by 8- to 9-year-old children suggested that the items were appropriate for children this age. However, future research with the MIMS-C or similar items should consider including covariates, such as reading comprehension, cognitive ability, or also emergent bilingual student status to account for potential differences in ability to understand grammatically complex statements.

Our study also measured mathematics intelligence mindset during one time point as a static belief. However, additional indicators are necessary in order to provide more robust evidence for the dimensionality and measurement of children’s intelligence mindsets in mathematics using growth, fixed, and mixed mindset items. These beliefs may look different if they were measured at another time of year. Thus, future research should consider test-reliability and within-person variability of mindsets held across different contexts and time points when assessing for the robustness of these models of intelligence mindset in the mathematics domain.

8.7.1. Generalizability of study findings

The generalizability of our study results is limited to the characteristics of our study sample (i.e., mathematics domain in children in third grade attending elementary schools within the southeast United States). There may be differences in the beliefs about the nature of mathematics ability from children who live in different regions of the U.S., who live in different countries with varying cultures, or who are younger or older. Responses on mixed mindset items may also be different for intelligence mindset more generally or in other school subjects. Mindsets are also researched in non-academic contexts and with other characteristics or traits, but our study findings do not examine non-intelligence mindsets about mathematics ability (e.g., music ability, Burgoyne et al., 2019; shyness, Valentiner et al., 2011).

As an example, while the responses on these new mindset items are not directly generalizable to samples of other ages, we do know that previous research has found varying degrees of skewness in growth mindset item responses in samples of college students (Dweck et al., 1995) and responses to items assessing more broad motivation variables in preschool children (i.e., Bridgeman & Shipman, 1978). There are various reasons for the level of skewness found in each age group. Some older research suggests that for college students, social desirability plays a role in the skewness of responses (Boyum, 1988; Dweck et al., 1995; Leggett, 1985), whereas more recent research does not find a role of social desirability in college student responses on growth mindset items (Dai & Cromley, 2014; Spinath & Stiensmeier-Pelster, 2003). Although we may expect to find skewness in these items in samples of students in other ages than third grade, it will be up to future research to determine whether similarities in responses exist in students of different ages and to identify the reasons for skewness in responses to these items for each age group as well.

Despite these generalizability limitations, our study results do provide an initial proof of concept that the broad construct of “mindset” or “implicit theories” can include nuanced beliefs that overlap between growth-oriented and fixed-oriented beliefs. Future research is needed to further investigate the existence of mixed mindsets in mathematics and broadly for intelligence, as well as mindsets for other specific areas of ability, other non-intelligence specific traits, across different developmental periods, and for people in other countries and cultures.

8.8. Conclusion

In summary, we provide empirical evidence to suggest that mathematics intelligence mindset is a multidimensional construct. This construct can include aspects of fixed mindsets, defined as rigid beliefs that mathematics intelligence cannot change, and aspects of growth mindsets, which can include the traditional rigid growth beliefs as well as more nuanced mixed mindset ideas that acknowledge fixed components related to growth in ability. We suggest an expansion to the previously theorized definition of growth intelligence mindset in mathematics that includes these more nuanced beliefs about the capacity to change ability over time and individual differences in abilities and the change that can happen. This expansion may more precisely characterize the beliefs about the malleability of mathematics intelligence that children hold and provide educators and parents with messages that can potentially be more encouraging to children and are more strongly linked with mathematics achievement than growth mindset alone. Additionally, we also provide a new measure that can be used to assess the multidimensional components of mathematics intelligence mindset in third-grade students. While there is much more research to be done to fully understand the full range of mathematics intelligence mindsets in children, the results from the current study provide a future avenue for research to take in the study of intelligence mindsets.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that support the findings of this study are available from the corresponding author, Connie Barroso, upon request. The data are part of a larger study for which the data will be publicly available in 2024. These data will be linked through the OSF page when available.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cedpsych.2023.102179.

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