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RESEARCH PAPER – MATHEMATICS EDUCATION

Investigating the association between students' strategy use and mathematics achievement

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Abstract
This observational study used data from 270 second-grade students to investigate the association between students' strategy use for multidigit addition and subtraction and their mathematics achievement. Based on strategies they used during a mathematics interview, students were classified into the following strategy groups: (a) standard algorithm, (b) invented, (c) mixed, and (d) unclassified. We used two-level hierarchical linear regression to investigate the association between students' strategy use and their performance on a standardized test in mathematics. Results indicated that students in the mixed strategy groups had significantly higher mathematics achievement than those in the standard algorithm and the unclassified groups.

KEYWORDS
elementary mathematics, mathematics achievement, multidigit addition and subtraction, single-digit addition and subtraction, student strategies

1 | INTRODUCTION

Whole-number arithmetic is a main focus of the mathematics curriculum in the early years of elementary education, and appropriate learning experiences in these grades improve students' chances for later success (National Research Council, 2001). Recent reform efforts in mathematics education attempt to create space for students to use problem-solving strategies efficiently, creatively, and flexibly (National Council of Teachers of Mathematics (NCTM), 1989; 2000; National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA and CCSSO), 2010; Peters et al., 2013).

Some scholars have argued that when students are encouraged to invent their own strategies for solving addition and subtraction problems, they develop better understandings of related mathematics concepts and perform better on tests of their mathematical abilities than those who use standard algorithms (Carpenter et. al, 1998; Kamii & Dominic, 1998). Cobb and Wheatley (1988) reported that many students who correctly carry out standard algorithms procedurally do not understand the reasons for the procedures or the underlying concepts. Nevertheless, many students continue to be introduced to the standard algorithms for addition and subtraction by the time they are in second grade (Sahin, 2015).

Some researchers have claimed that the strategies students use to solve mathematics problems can be influenced by the class environment in which they learn mathematics (Cobb et al., 1992; Torbeyns et al., 2009; Yackel & Cobb, 1996). Students who receive mathematics instruction focused on application of one particular strategy are likely to use this strategy to solve mathematical tasks.

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Moreover, those who receive mathematics instruction focused on the development of various student-invented strategies from the start of their mathematics instruction tend to exhibit more variation in formal and informal strategies used while solving mathematics problems (Torbeyns et al., 2009).

The purpose of the present study was to perform an empirical study of the extent to which second-grade students used various strategies for solving multidigit computation problems and to investigate the association between students' strategy use and their performance on mathematics achievement tests. We focused on second grade in this study, because whole-number addition and subtraction are strongly emphasized in the second-grade curriculum, and we know that many students have been introduced to the standard algorithm by this time. Extant studies comparing students' use of invented strategies and standard algorithms have been mostly small in scale and qualitative or descriptive in nature. In this study, we used empirical data to investigate students' strategy use for multidigit addition and subtraction and their mathematics achievement at a much larger scale than previous studies have done. The present study was guided by the following research questions.

1. What strategies do second-grade students use to solve multidigit addition and subtraction problems?
2. To what extent are the problem-solving strategies used by second-grade students associated with their performance on mathematics tests?

2 | LITERATURE REVIEW

Extensive research has been conducted regarding student-invented strategies for solving multidigit addition and subtraction problems (Blöte et al., 2001; Carpenter et al., 2015; Fuson et al., 1997; Hiebert & Wearne, 1993; Torbeyns et al., 2006). These studies have identified a variety of invented strategies children use to solve multidigit addition and subtraction problems. Fuson et al. (1997) classified six types of student-invented strategies: (a) "the decompose-tens and-ones method: add or subtract everywhere and then regroup"; (b) "the decompose-tens and-ones method: regroup then add or subtract everywhere"; (c) "the decompose-tens and-ones method: alternate adding or subtracting and regrouping"; (d) "the begin-with-one-number method: begin with one and move up or down by tens and ones"; (e) "mixed methods: add or subtract tens, make sequence number with original ones, add or subtract other ones"; and (f) "the change both numbers methods" (pp. 147–148). Carpenter et al. (2015) identified three types of invented strategies: incrementing, combining the same units, and compensating. These three categories combine several categories that are presented separately by Fuson et al. (1997). Table 1 summarizes and gives examples of the three types of invented strategies described by Carpenter et al. (2015), including explanations from Fuson et al. (1997).

Although student-invented strategies are applicable to specific numbers and not necessarily generalizable, Carpenter et al. (2015) argue that they tend to reflect an intuitive or informal understanding of the underlying laws or properties of operations and equality. Invented strategies involve students making deliberate decisions about how to solve computation problems. Students' use of these invented strategies may indicate that they understand that numbers can be decomposed and recomposed in different ways, and that they can perform operations on multidigit numbers with understanding (Hiebert & Carpenter, 1992; Kamii & Livingston, 1994).

Besides invented strategies, students also learn standard algorithms to solve multidigit addition and subtraction problems. "An algorithm is a step-by-step process that guarantees the correct solution to a given problem, provided the steps are executed correctly" (Barnett, 1998, p. 69). Standard algorithms differ from student-invented strategies. Students are not likely to invent the steps involved with standard algorithms on their own. Therefore, we can assume students who use these strategies were instructed in the steps to execute the algorithm(s). Standard algorithms are useful, because they are highly generalizable and can be accurate and efficient, but their basis in the properties of number, operations, and equality are less obvious than many of the strategies students invent. This latter point is why we categorize them as instructed, not invented. Success or failure in carrying out an instructed algorithm yields more insight into whether a student knows how to carry out the procedures in the algorithm and less insight into his or her understanding of the underlying properties of numbers, operations, or equality.

Historically, the application of standard algorithms has been a primary emphasis in the mathematics curriculum at the elementary and secondary levels (Mingus & Grassl, 1998). The Common Core State Standards for Mathematics (CCSSM), however, emphasize the use of strategies that are based on place value and properties of operations in first and second grade, and strategies and algorithms based on place value and properties of operations in third grade. The CCSSM specify that students should "fluently add and subtract multidigit whole numbers using the standard algorithm" in fourth grade (NGA & CCSSO, 2010, p. 29).

2.1 | Theoretical framework

Murray and Olivier (1989) formulated a theoretical framework that describes four levels of children's understanding of two-digit numbers, which is summarized in Table 2.

They asserted that level-three understanding provides children with the conceptual basis to use invented strategies

TABLE 1 Examples of student-invented strategies

Strategies	28 + 25 = □	53 − 28 = □	28 + □ = 53
Incrementing	Count on/add on tens, then ones	Count down/subtract tens, then ones	Count on/add up tens, then ones
	28, 38, 48, 49, 50, 51, 52, 53	53, 43, 33, 32, 31, 30, 29, 28, 27, 26, 25	28, 38, 48, 49, 50,51, 52, 53:25 added
	28 + 20 → 48 + 5 → 53	53 − 20 → 33 − 8 → 25	28 + 20 → 48 + 5 → 53:25 added
	Count on/add on to make a 10, count on/add on tens, then the rest of ones	Count down/subtract to make a 10, count down/subtract tens, then the rest of the ones	Count up/add up to make a 10, count up/add up tens, then the rest of ones
	28, 29, 30, 40, 50, 51, 52, 53	53, 52, 51, 50, 40, 30, 29, 28, 27, 26, 25	28, 29, 30, 40, 50, 51, 52, 53:25 added
	28 + 2 → 30 + 20 → 50 + 3→ 53	53 − 3 → 50 − 20 → 30 − 5 → 25	28 + 2 → 30 + 20 → 50 + 3 → 53:25 added
	Count on/add on tens, add on ones, count on/add on other ones	Count down/subtract tens, add original ones, count down/subtract other ones	Count up/add up tens, add original ones, count up/add up other ones
	20, 30, 40, 48, 49, 50, 51, 52, 53	50, 40, 30, 33, 32, 31, 30, 29, 28,27, 26, 25	20, 30, 40, 48, 59, 50, 51, 52, 53:25 added
	20 + 20 → 40 + 8 → 48 + 5 → 53	50 − 20 → 30 + 3 → 33 − 8 → 25	20 + 20 → 40 + 8 → 48 + 5 → 53:25 added
Combining the Same Units	Add tens, add ones, combine tens and ones	Subtract tens, subtract ones, combine totals	**
	20 + 20 = 40, 8 + 5 = 13, 40 + 13 = 53	50 − 20 = 30, 3 − 8 = −5, 30 − 5 = 25	
	Add ones, add tens, combine tens and ones	Subtract ones, subtract tens, combine totals	
	8 + 5 = 13, 20 + 20 = 40, 13 + 40 = 53	3 − 8 = −5, 50 − 20 = 30, 30 − 5 = 25	**
Compensation	Overshoot and come back	Overshoot and come back	If it were 30, 30 + 23 would be 53, but it is 28, so add 2 more to 23, it would be 25
	30 + 25 would be 55. 55 − 2 would be 53	53 − 30 would be 23, 23 + 2 would be 25	Make initial number a tens number, change other to maintain difference
	Move some from one number to the other to make a tens number	Make subtracted number a tens number, change other to maintain the difference	28 + □ = 53 is the same as 30 + □ = 55,
	28 + 2 → 30, 25 − 2 → 23, 30 + 23 → 53	53 − 28 = 55 − 30 = 25	so □ = 25

Note: Adapted from "Children's Conceptual Structures for Multidigit Numbers and Methods of Multidigit Addition and Subtraction," by K. Fuson, D. Wearne, J.C. Hiebert, H. G. Murray, P. G. Human, A.I. Olivier, T. P. Carpenter, E. Fennema, 28, pp. 147–148. Copyright 1997 by The National Council of Teachers of Mathematics, Inc.

TABLE 2 Description of children's levels of understanding of two-digit numbers

Levels of understanding of two-digit numbers	Description
One	A child can count a number of objects and has the knowledge of the number names and their associated numerals, however does not assign meaning to the individual digits
Two	A child can conceptualize a given number as an abstract unit item with a meaning and does not need physical referents
Three	A child can see two-digit numbers as composite units of decades and ones
Four	A child can see two-digit numbers as groups of tens and some ones

and level-four understanding facilitates a progressive shortening and abstraction of those strategies. They suggested that level-four understanding is a prerequisite to execute a standard algorithm meaningfully. They argued that when children performing at level-one and level-two have difficulty in computation with larger numbers, teachers tend to "help" them by introducing the standard algorithm. Murray and Olivier (1989) further claimed that even if the teachers try to build a conceptual basis for the algorithms, such efforts would be ill-fated if levels two and three are bypassed in the development of children's mathematical understanding. They concluded that facility in executing the algorithm could hide serious deficiencies in children's understanding.

The framework developed by Fuson et al. (1997) provides a sequential development of students' understanding of multidigit English number words (such as 54) and written number marks (54). The framework consists of five levels of conceptual structures of two-digit numbers that students acquire. These conceptual structures are: (a) Unitary, (b) Decade and ones, (c) Sequence tens and ones, (d) Separate tens and ones, and (e) Integrated sequence-separate tens and ones.

In the unitary level, students are not able to differentiate quantities into groupings, and number words and number marks into parts. According to a student at this level, the 1 in 18 is not related to the teen in 18, and 18 is not separable into 10 and 8. The decade and ones level requires students to be able to separate the decade and the ones parts of a number word and begin to relate each part to which the quantity refers. A student at this level understand that in 53 50 refers to 50 objects and three to three objects.

The sequence-tens and ones level requires children to construct a 10-structured version of the decade and ones conception. At this level, children are able to count by tens, see the groups of tens within a quantity, and choose to count these groups by tens (e.g., "ten, twenty, thirty, forty"). The separate-tens and ones conception requires children to see the quantity as separate tens and ones. In this stage, students are able to see and count the groups rather than the objects in the groups (e.g., "one ten, two tens, three tens, four tens").

The integrated sequence-separate tens and ones level requires constructing both the sequence tens and ones and separate tens and ones conception and being able to use them interchangeably based on the problem structures. A child at this level is able to recognize immediately that 60 has six tens without counting by tens to 60 with keeping track of how many tens he counted or counting six tens to find out that they make 60.

Students' construction of these conceptual structures depends on their experiences both in and out of school (Fuson et al., 1997). Therefore, not all students construct all the conceptions (Verschaffel et al., 2007). Students' construction of these conceptual structures of multidigit numbers affects their use of different strategies for multidigit addition and subtraction problems (Carpenter et al. (2015). The current study was informed by these two theoretical frameworks developed for students' understanding of multidigit structures of the numbers.

3 | METHODS

This study uses a cross-sectional design to investigate the association between second-grade students' strategy use and mathematics achievement. Students were nested in schools. Because unobserved school-level factors may exert an influence on students' strategies and performance on the mathematics test, we used hierarchical linear modeling (HLM) to account for the nested structure of the data (Gall et al., 2007).

3.1 | Participants and data sources

The sample for the present study included 270 second-grade students from 22 public elementary schools located in two adjacent school districts in Florida. Teachers and students in the 22 schools were participating in a larger study investigating the impact of teachers' opportunities to participate in a professional development program based on Cognitively Guided Instruction (CGI) on student mathematics achievement (Schoen et al., 2020). The present study used student data that were gathered in the 1st year of the larger study. About 52% of the students in this sample were females and 48% were males. Approximately 38% of the students were White, 28% were Hispanic, and 14% were African American. The rest of the ethnicities (e.g., Asian, American Indian) made up approximately 10% of the sample, and ethnicity was not indicated for about 10% of the students in the sample. [Article updated on October 3, 2020 after first publication: This paragraph was updated to reflect the correct year of publication for Schoen et al.]

The data for the present study were drawn from a student mathematics interview named Mathematics Performance and Cognition (MPAC; Schoen et al., 2016), and the Iowa Test of Basic Skills-Math Problems test (ITBS; Hoover et al., 2001). The ITBS and MPAC interviews were administered in spring 2014. This study used the ITBS test to measure mathematics achievement, and the MPAC interviews to determine students' strategy use. The MPAC interviews and ITBS tests were administered by trained research personnel. Students were provided with base-10 blocks, snap cubes, paper, and markers during the interviews. They were instructed by the interviewers to solve the problem in any way that made sense to them. Thirteen percent of the interviews were coded by two independent raters, and the percentage-agreement method was used to calculate the proportion of interrater agreement for the major strategy type, which was found to be 83% (Schoen et al., 2016).

3.2 | Data analysis

3.2.1 | Describing and classifying students' strategy use

Seven multidigit problems (four word problems and three computation problems) drawn from the MPAC interviews were used to describe and classify students' strategy use (see Table 5 in the results section for problem types and number combinations used in the problems). To describe the strategies students used for each problem we classified students' strategies as: (a) unitary (i.e., modeling with/counting by ones), (b) modeling with tens, (c) invented strategies, (d) standard algorithm, and (e) other strategies. A strategy was classified as "other" when it did not fit into any of the named categories. Student responses were classified in accordance with the strategy they used, regardless of whether they obtained a correct answer.

Next, we used the strategies used by individual students for seven problems to classify individual students into one of four mutually exclusive and collectively exhaustive strategy groups to be used in the regression analyses. The strategy groups were named as follows: (a) standard algorithm, (b) invented, (c) mixed, and (d) unclassified. Table 3 lists the strategy groups and their descriptions (see Table 6 in the results section for descriptive statistics about the strategy groups). These categories are nominal and do not imply any sort of a priori ranking in our study.

3.2.2 | Multilevel modeling

For inferential statistics, we used HLM 7.03 Student Version (Raundenbush et al., 2017) to analyze the data using two-level hierarchical-linear regression to determine the association of strategy groups with the mathematics achievement. We used a two-level model with students nested in schools. The standardized score from the ITBS-Math Problems test was the dependent variable, and strategy use was the independent variable. The strategy-use variable had four categories, so it was dummy coded. This created four variables which we named as standard algorithm, invented, mixed, and unclassified.

We ran three models to see which strategy group differed significantly from the other strategy groups. In the first model, the dummy variable corresponding to standard algorithm was the reference variable. In the second model, the dummy variable corresponding to invented was the reference variable. In the third model, the dummy variable corresponding to mixed was the reference variable. Using three models, we were able to compare all strategy groups to each other. Table 4 shows the regression equations for the three models.

4 | RESULTS

Table 5 shows the percentages of strategies students used for each of the seven problems. As Table 5 shows, students used the standard algorithm most often which was followed by the unitary strategies, and that was followed by the invented strategies. Students used modeling with tens strategy the least. Among all seven problems, the computation problem *201 − 199 = ?* was the most difficult for the students, with only 25% of the interviewed students providing a correct answer. The most common strategy used by the students for this problem was the standard algorithm. The standard algorithm-based

TABLE 3 Strategy group descriptions

Strategy group	Description
Standard algorithm	The student was observed to use (or attempted to use) the standard U.S. algorithm to add or subtract and was not observed using any invented strategies
Invented	The student was observed using an invented strategy at least one time and was not observed using any standard algorithm
Mixed	The student was observed using an invented strategy at least one time and using (or attempting to use) a standard algorithm at least one time
Unclassified	The student was not observed using any invented strategy or standard algorithm

TABLE 4 Hierarchical linear regression models

Models	Hierarchical linear regression equation
Model 1	$\mathrm{ITBS\_MAT} = \gamma_{00} + \gamma_{10} * \mathrm{UNCLASSI} + \gamma_{20} * \mathrm{INVENTED} + \gamma_{30} * \mathrm{MIXED} + u_0 + r$
Model 2	$\mathrm{ITBS\_MAT} = \gamma_{00} + \gamma_{10} * \mathrm{UNCLASSI} + \gamma_{20} * \mathrm{ALGORITH} + \gamma_{30} * \mathrm{MIXED} + u_0 + r$
Model 3	$\mathrm{ITBS\_MAT} = \gamma_{00} + \gamma_{10} * \mathrm{UNCLASSI} + \gamma_{20} * \mathrm{ALGORITH} + \gamma_{30} * \mathrm{INVENTED} + u_0 + r$

TABLE 5 Problems from the mathematics interviews and students' strategies

Problem type	Number combination	% of Strategies used by the students				
		Unitary	Modeling with tens	Invented	Standard algorithm	Other
Word problem—join result unknown	(28, 43)	14.3%	13.3%	15%	52.4%	4.9%
Word problem—join change unknown	(17, 26)	45.8%	3.8%	8.4%	33.6%	8.4%
Word problem—join result unknown	(49, 56)	8.4%	13.3%	15%	52.8%	10.5%
Word problem—separate change unknown	(42, 36)	30.8%	2.1%	9.1%	40.9%	17.1%
Computation problem	63 − 17 = ?	29%	8%	10.8%	45.8%	5.6%
Computation problem	100 − 3 = ?	60.5%	5.6%	16.4%	7.7%	7%
Computation problem	201 − 199 = ?	11.9%	1.7%	10.5%	54.5%	19.6%

TABLE 6 Sample means and standard deviations for ITBS math problems standardized score for each strategy group

Strategy group	*n*	Mean	Std. Deviation
Mixed	63	189.1	18.9
Invented	31	181.9	18.6
Standard algorithm	148	174.5	18.9
Unclassified	28	159.4	17.7

solution to this problem involves two regrouping processes, which may have contributed to its difficulty. The standard algorithm-based solution to the problem *100 − 3 = ?* also includes two regrouping processes for the two zero digits in the minuend, but this item was the easiest problem for the students, with 81% of the interviewed students providing a correct answer. This may be due to the subtrahend being three, which makes it easy for students to count down by ones. As Table 5 shows, this counting-by-ones strategy was the most prevalent strategy used by the students for this problem.

We next classified the students into the standard algorithm, invented, mixed, and unclassified strategy groups using the criteria given in Table 3. Based on the given criteria, 148 students were classified into the standard algorithm group, 31 into the invented group, 63 into the mixed group, and 28 into the unclassified group.

Table 6 shows the mean and standard deviation of the ITBS-Math Problems scores for students in each of the strategy groups. The mixed strategy group had the highest mathematics achievement (M = 189.1, SD = 18.9). The invented strategy group had the second highest (M = 181.9, SD = 18.6), and the standard algorithm group had the third highest (M = 174.5, SD = 18.9) mathematics achievement. The unclassified group had the lowest mathematics achievement (M = 159.4, SD = 17.7).

We used three multilevel regression models to determine the association of strategy groups with mathematics achievement and provide the results of the models in Table 7. Students in the standard algorithm, the invented strategy, and the mixed strategy groups performed higher on the ITBS-Math Problems test than the students in the unclassified group (ps < .001). The students in the standard algorithm group had lower mathematics achievement than the students in the invented strategy group (p = .060) and in the mixed strategy group (p < .001). The students in the invented strategy group had lower mathematics achievement than the students in the mixed group (p = .091), but the p value was highest for this latter comparison.

5 | DISCUSSION

The CCSSM emphasizes the use of strategies based on number properties in early elementary grades, and previous research reported the benefits of students using strategies of their own choosing/invention (Carpenter et al., 1998; Cobb & Wheatley, 1988; Kamii & Dominic, 1998). In our study, only 34% of second-grade students used an invented strategy at least one time during the interviews, and 78% of the students used a standard algorithm at least once. The reason that a majority of students did not use an invented strategy during the mathematics interview could be simply because they did not prefer to use an invented strategy or because they had not yet acquired level-three or level-four understanding, which according to Murray and Oliver (1989) must be acquired to be able to use invented strategies with understanding.

The results of the current study showed that students who primarily used the standard algorithm had statistically significantly lower mathematics achievement than the students who used a combination of invented strategies and standard algorithms (i.e., the mixed group). The standard algorithm group had lower mathematics achievement than the students in the invented strategy group (p = .060), and the invented strategy group had lower mathematics achievement than the students in the mixed group (p = .091). This reveals that the use of invented strategies is associated with higher achievement than the use of standard algorithms, and it implies that

TABLE 7 Combined results of the three regression models

Reference group	Strategies compared to reference group	Coefficients	Standard Error	*t*-ratio	Approx. *df*	*p* value
Standard algorithm	Unclassified	−13.605	3.561	−3.821	245	<.001
	Invented	6.497	3.433	1.893	245	.060
	Mixed	12.848	2.599	4.943	245	<.001
Invented	Unclassified	−20.102	4.583	−4.386	245	<.001
	Standard algorithm	−6.497	3.433	−1.893	245	.060
	Mixed	6.351	3.747	1.695	245	.091
Mixed	Unclassified	−26.453	3.992	−6.626	245	<.001
	Standard algorithm	−12.848	2.599	−4.943	245	<.001
	Invented	−6.351	3.747	−1.695	245	.091

encouraging students to use strategies of their own invention to solve problems may serve to increase their performance on standardized mathematics tests. This recommendation goes against the grain of the conventional approach to mathematics instruction currently in use in many second-grade classrooms, where students are commonly instructed on the application of standard algorithms in place of using strategies of their own invention.

The theoretical framework offered by Murray and Oliver (1989) may explain why the students who used a combination of invented strategies and standard algorithm had the highest mathematics achievement of the strategy groups. They suggested that level-four understanding of two-digit numbers facilitates the shortening and abstraction of the invented strategies, and it is also a prerequisite to employ the standard algorithm meaningfully. It may be the case that students who used a combination of invented strategies and standard algorithm had a level-four understanding of two-digit numbers, hence had a better understanding of the standard algorithm, which yielded less errors in computation and higher mathematics achievement on the test. It could also be the case that having an understanding of both invented strategies and the standard algorithm provided more flexibility and enabled students to choose the most efficient strategies for specific problems. For example, an invented strategy could be more efficient than the standard algorithm for $101 - 92$ (a student may say "I know $92 + 8$ is 100 and then, $+1$ is 101, so $101 - 92$ is 9"), whereas the standard algorithm can be more efficient than an invented strategy (such as incrementing or compensation) to evaluate $198 - 47$, where regrouping is not necessary to perform the algorithm, and counting and other invented strategies are not applied as easily.

6 | LIMITATIONS

This study had some limitations that should be noted. First, we had a relatively low number of multidigit items that were used to determine students' strategy use. Second, if a student was not observed using a given strategy, we cannot infer the student was not able to use that type of strategy. The student might have simply decided not to use strategies other than the ones we observed the student using. Third, the study design is correlational, so the causal relations and the direction of their influence should not be implied. The current study is cross sectional, and provides insight into students' use of strategies in second grade, but it cannot provide longitudinal perspective on changes in strategy use or performance over time, which is worthy of exploration in future studies. Finally, the sample was formed by convenience, and an a priori power analysis was not done to determine the optimal sample size. This factor of the design should be considered when interpreting *p* values and point estimates. Moreover, while the statistical analyses may account for some of the unobserved localized effects in this sample, the extent to which these results would be replicated or would generalize to the broader population is not currently known.

7 | CONCLUSION

In this study, we found that second-grade students who used a combination of invented strategies and standard algorithms had higher performance on standardized tests in mathematics than those who only used standard algorithms to solve multidigit addition and subtraction problems. This large-scale, empirical study provides new evidence to support theory and previous empirical results that have been found at a smaller scale reporting the benefit of children using invented strategies. We recommend that future research should involve conducting more interviews to determine whether students who use standard algorithm know and explain conceptually what they are doing or if they execute the steps of the algorithm mechanically.

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REFERENCES

Barnett, H. J. (1998). A brief history of algorithms in mathematics. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics* (pp. 69–77). National Council of Teachers of Mathematics.

Blöte, A. W., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology*, *93*(3), 627.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2015). *Children's mathematics: Cognitively guided instruction*. Heinemann.

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education*, *29*(1), 3–20.

Cobb, P., & Wheatley, G. (1988). Children's initial understandings of ten. *Focus on Learning Problems in Mathematics*, *10*(3), 1–28.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, *23*(1), 2–33.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, *28*, 130–162.

Gall, M. D., Gall, J. P., & Borg, W. R. (2007). *Educational research* (8th ed.). Pearson.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 65–98). New York, NY: Macmillan.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, *30*(2), 393–425.

Hoover, H. D., Dunbar, S. B., & Frisbie, D. A. (2001). *Iowa tests of basic skills (ITBS) forms A, B, and C*. Riverside Publishing Company.

Kamii, C., & Dominick, A. (1998). The harmful effects of algorithms in grades 1–4. *The Teaching and Learning of Algorithms in School Mathematics*, *19*, 130–140.

Kamii, C., & Livingston, S. J. (1994). *Young children continue to reinvent arithmetic–3rd grade: Implications of Piaget's theory*. Teachers College Press.

Mingus, T. T., & Grassl, R. M. (1998). Algorithmic and recursive thinking: Current beliefs and their implications for the future. In *The teaching and learning of algorithms in school mathematics* (pp. 32–43). : National Council of Teachers of Mathematics.

Murray, H., & Olivier, A. (1989). A model of understanding two-digit numeration and computation. In G. Vergnaud, J. Rogalski, & M. Artique (Eds.), *Proceedings of the thirteenth international conference for the psychology of mathematics education* (Vol. *3*, pp. 3–10). Paris.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Author.

National Governors Association (NGA) Center for Best Practices & Council of Chief State School Officers (CCSSO). (2010). *Common core state standards for mathematics*. : Author.

National Research Council, & Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Peters, G., De Smedt, B., Torbeyns, J., Ghesquière, P., & Verschaffel, L. (2013). Children's use of addition to solve two-digit subtraction problems. *British Journal of Psychology*, *104*(4), 495–511.

Raudenbush, S. W., Bryk, A. S., & Congdon, R. (2017). *HLM 7.03 for Windows* [Computer software]. : Scientific Software International, Inc.

Sahin, N. (2015). *The effect of cognitively guided instruction on students' problem solving strategies and the effect of students' use of strategies on their mathematics achievement* (Unpublished Doctoral Dissertation). University of Central Florida.

Schoen, R. C., LaVenia, M., Champagne, Z., & Farina, K. (2016). *Mathematics Performance and Cognition (MPAC) Interview: Measuring first- and second-grade student achievement in number, operations, and equality in spring 2014* (Report No. 2016–01). : Florida State University. https://doi.org/10.17125/fsu.1493238156

Schoen, R. C., LaVenia, M., Tazaz, A., Farina, K., Dixon, J. K., & Secada, W. G. (2020). Replicating the CGI experiment in diverse environments: Effects on Grade 1 and 2 Student Mathematics Achievement in the First Program Year (Research Report No. 2020. (R)02). Florida State University [Article updated on October 3, 2020 after first publication: This reference was updated to reflect the correct report no.]

Torbeyns, J., De Smedt, B., Ghesquière, P., & Verschaffel, L. (2009). Acquisition and use of shortcut strategies by traditionally schooled children. *Educational Studies in Mathematics*, *71*(1), 1–17. https://doi.org/10.1007/s10649-008-9155-z

Torbeyns, J., Verschaffel, L., & Ghesquiere, P. (2006). The development of children's adaptive expertise in the number domain 20 to 100. *Cognition and Instruction*, *24*(4), 439–465. https://doi.org/10.1207/s1532690xci2404_2

Verschaffel, L., Grrer, B., & De Corte, E. (2007). Whole numbers concepts and operations. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557–628). Information Age Publishing.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*, 458–477. https://doi.org/10.2307/749877

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