

# M-CLIPS

Mathematics – Cognition, Language, Interaction, Problem Solving

Version 3.0

COVER PAGE – COMPLETE FOR EACH LESSON AND ATTACH TO FIELD NOTES  
AND RUBRICS SHEET

## Background Information

Video ID: \_\_\_\_\_ Date: \_\_\_\_\_

Observer: \_\_\_\_\_ Grade: \_\_\_\_\_

Length of observed lesson: \_\_\_\_\_

## Standards Domains (select all that apply)

Numbers and Operations

Algebraic Reasoning

Geometry

Measurement

Data Analysis & Probability

Other

## Notes:

## M-CLIPS Rating Sheet - INDIVIDUAL

COMPLETE THIS RATING SHEET AS SOON AS POSSIBLE AFTER EACH OBSERVATION,  
BASED ON YOUR OWN INDIVIDUAL OBSERVATIONS.  
PLACE WITH YOUR FIELD NOTES.

Video ID: \_\_\_\_\_ Observer: \_\_\_\_\_

### Principle 1: Problem solving plays an integral role in the teaching and learning of mathematics.

Rubrics	Rating	Notes
Autonomy (AUTO)		
Variation (VAR)		
Respect (RSPT)		
Cognitive Complexity (CC)		

### Principle 2: The teacher attends to students' mathematical thinking.

Rubrics	Rating	Notes
Attend (ATND)		

### Principle 3: Students engage with their peers' ideas about mathematics.

Rubrics	Rating	Notes
Teacher Support for Peer Interaction (TSPI)		
Peer Interaction (PRI)		

### Principle 4: The teacher supports students' understanding and meaningful use of the language of mathematics.

Rubrics	Rating	Notes
Express-Elaborate (EE)		
Connecting Representations (CR)		

# Principle 1: Problem solving plays an integral role in the teaching and learning of mathematics.

<b>Autonomy (AUTO)</b> <b>The teacher facilitates students' use of their emerging understandings of mathematical ideas to devise <i>their own</i> ways to solve mathematics problems.</b>	
<b>High (4, 5)</b> <i>Strong to expert implementation of principle</i>	<p>The teacher consistently creates and sustains opportunities for students to solve mathematics problems in any way they can and that makes sense to them.</p> <p>All of the following are present:</p> <ol style="list-style-type: none"> <li>Substantive part(s) of the lesson involve students in attempting to solve problems that the teacher has not shown them how to solve.</li> <li>The teacher refrains from telling or showing students how they <i>should</i> solve problems, including when students experience difficulty.*</li> <li>All students have opportunity to solve problems in their own ways (and decide how they will try to solve the problem).</li> <li>The pace of instruction affords sufficient time for students to solve problems using strategies with understanding.</li> </ol> <p>*The teacher responds to student difficulties in ways that stimulate rather than dominate student thinking—for example, by helping students to understand the problem, reflect on their approach, or by making modifications to the problem that bring the mathematics closer to the students' understanding (e.g., reframe in meaningful context, modify numbers). The students remain in control of devising <i>their own ways</i> to solve problems.</p>
<b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i>	<p>The teacher encourages or creates some opportunities for students to solve problems in ways of their own choosing, but those opportunities are limited.</p> <p>The High rating is not warranted for one or more of the following reasons:</p> <ol style="list-style-type: none"> <li>A substantial portion of the lesson involves explicit instruction on and/or practice of specific strategies for solving certain types of problems.</li> <li>The teacher allows flexibility for some students to solve problems in their own ways, while others are expected to solve problems in prescribed ways.</li> <li>The teacher provides insufficient time or tools for students to make sense of the problem.</li> <li>The teacher invites students to solve problems by selecting a strategy from a menu of options that were previously instructed by the teacher or curriculum materials.</li> <li>The teacher tells the students that they can use any strategy they want to use but subsequently directs them to use specific strategies that are preferred by the teacher.</li> <li>Teacher response to student struggle tends to direct or heavily guide students toward the teachers' thinking (rather than stimulating the students' thinking).</li> </ol>
<b>Low (0, 1)</b> <i>Contradicts principle</i>	<p>The teacher does not encourage students to solve problems by using strategies of their own choosing or invention. The teacher consistently shows or tells students how to solve problems (i.e., explicit instruction) and expects students to practice and use the instructed strategy.</p> <p>The teacher directs students to use a particular solution method that is known, prescribed, or heavily prompted by the teacher or textbook. A small amount of divergent thinking may be tolerated, such as when it involves ways to complete intermediate steps in an overall strategy. (1)</p> <p>The teacher insists that students use a specific way of solving a given problem or actively discourages invented strategies or other forms of divergent thinking. Independent thinking or alternative methods of solving a problem are ignored, discouraged, or marginalized. (0)</p>

# Principle 1: Problem solving plays an integral role in the teaching and learning of mathematics.

<b>Variation (VAR)</b> <b>Students exhibit variation and individual differences in their thoughts and approaches to solving mathematics problems.</b> <i>Variation in students' approaches to solving problems can be observed in the tools and strategies students use and/or the ways in which students employ particular tools or strategies. The presence or use of manipulatives or other physical materials are not required for medium or high ratings. Ratings should include the ways students are solving problems when they are working individually or in small groups, not just what is made public during whole-group discussions.</i> <i>Individual differences in thought and mathematical understanding are natural and omnipresent in every group of persons. This rubric seeks to observe whether these differences manifest in observable features in the strategies that students are using to solve mathematics problems in the classroom.</i> <i>This is one of the rubrics that focuses on what students do—not what the teacher does—so the language focuses on variation in students' approaches to solving mathematics problems.</i>	
<b>High (4, 5)</b> <i>Strong to expert implementation of principle</i>	Individual differences in thought and mathematical understanding present as widespread, substantive variation* in students' approaches to solving mathematics problems. Individual students use problem-solving strategies that they "own" and that make sense to them. *Variation may exist within or between their use of representations, strategies, or tools.
<b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i>	There is some variation in students' use of representations, strategies, or tools when they are solving problems independently or in small groups. The High rating is not warranted for one or more of the following reasons: <ol style="list-style-type: none"> <li>Variation in student approaches to solving problems appears to be constrained to a set of strategies that are expected to be carried out in particular ways.</li> <li>Variation in students' use of representations, strategies, and tools is only present in a small portion of the lesson or is limited to a small number of students.</li> </ol>
<b>Low (0, 1)</b> <i>Contradicts principle</i>	There is a high degree of uniformity in the representations, strategies, and tools used by students when they are solving problems. Students generally solve mathematics problems using the same representations, strategies, and tools. Small amounts of variation are limited to details in the enactment of intermediate steps within a prescribed strategy (e.g., while using the standard algorithm for adding multidigit numbers, students use varied strategies—counters, drawings, mental calculation—to add numbers in a given column). (1) Student errors in executing a highly prescribed or uniform procedure are the only source of variation, or the lesson offers limited or no opportunity for students to work on problems without the teacher's direct involvement. (0)

**Principle 1: Problem solving plays an integral role in the teaching and learning of mathematics.**

<p><b>Respect (RSPT)</b></p> <p><b>The teacher demonstrates respect and appreciation for each and every student’s abilities, perspectives, and contributions when they are solving mathematics problems.</b></p>	
<p><b>High (4, 5)</b> <i>Strong to expert implementation of principle</i></p>	<p>The teacher’s words and actions signal to the class that every student is capable and has something important and valuable to contribute to their own mathematical learning and that of the class. The teacher’s words and actions are characterized by the following:</p> <ol style="list-style-type: none"> <li>a. Respect and appreciation for each individual student’s abilities, perspectives, and contributions when solving math problems.*</li> <li>b. Emphasis on effort and initiative to learn through problem solving (e.g., teacher praises/values students’ efforts to make sense of mathematics and problem solve rather than their speed at producing correct answers).</li> <li>c. Confidence in students’ abilities as problem solvers, especially in moments of struggle (e.g., “Keep thinking about that”).</li> <li>d. (If applicable) Errors and misconceptions that surface publicly are positioned as productive contributions. They are not routinely suppressed, dismissed, or ignored.</li> </ol> <p>*The teacher may highlight and spend more instructional time on certain students’ strategies while also making clear that all students strategies and efforts are valued.</p>
<p><b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i></p>	<p>The teacher signals respect for students’ mathematical abilities, perspectives, and contributions; but respect for each and every student is not demonstrated at all times.</p> <p>The High rating is not warranted for one or more reasons:</p> <ol style="list-style-type: none"> <li>a. The teacher occasionally positions one or more students as having superior ability or ideas than their classmates.</li> <li>b. The teacher repeatedly interrupts or cuts some students short when they are trying to express their own ideas.</li> <li>c. Emphasis is on working quickly and complying with teacher directives, and/or there is a lack of emphasis on effort and initiative to learn through problem solving.</li> <li>d. The teacher conveys lack of confidence in some students’ abilities as problem solvers.</li> <li>e. Errors/misconceptions that surface publicly tend to be suppressed, dismissed, or ignored. They are rarely or never positioned as opportunities for learning.</li> </ol>
<p><b>Low (0, 1)</b> <i>Contradicts principle</i></p>	<p>The teacher implicitly or explicitly dismisses or devalues students’ mathematical abilities, perspectives, or contributions.</p> <p>The teacher consistently conveys that the teacher’s or textbook’s perspective is the superior (or uniquely correct) way of interpreting and solving mathematics problems and rarely or never invites students to examine alternative perspectives or default assumptions. (1)</p> <p>The teacher dismisses or devalues students’ ideas, perspectives, or efforts by only acknowledging, affirming, or celebrating student perspectives or ideas when they match those of the teacher. (1)</p> <p>The teacher explicitly dismisses/devalues students and/or their ideas/efforts by actively shaming them or by insisting that (mathematically correct) ideas that deviate from those of the teacher are inferior or incorrect. (0)</p>

# Principle 1: Problem solving plays an integral role in the teaching and learning of mathematics.

## Cognitive Complexity (CC)

**Enacted mathematical tasks engage students in high levels of cognitive complexity.**

*The students' role in working on the task(s) involves some or all of the following: creativity, reasoning or argumentation, planning, mathematical modeling, generalizations, explanations, justifications, or analysis of mathematical ideas. Such tasks cannot be solved simply by mechanical thinking, application of a predetermined or prescribed algorithm, or by recall of information from long-term memory.*

<p><b>High (4, 5)</b> <i>Strong to expert implementation of principle</i></p>	<p>Enacted tasks engage students at a high level of cognitive complexity through much of the lesson*.</p> <p>All of the following are present:</p> <ol style="list-style-type: none"> <li>The teacher assigns one or more tasks that engage students at a high level of cognitive complexity.*</li> <li>Assigned tasks offer appropriate challenge for most or all students in the class (not intractable, not trivial).</li> <li>The teacher's interactions with students maintain or increase the cognitive complexity of the task(s). This includes interactions before, during, and after work on tasks.</li> <li>The teacher positions at least one task as a mechanism for further examination or extension of important mathematical ideas. Arriving at an answer does not signal the end of the task. Rather, it signals the beginning of explanation, discussion, reflection, and/or analysis.</li> </ol> <p>*A task may have a high cognitive complexity from the point of assignment, or a task that initially appears to be moderate complexity is enacted such that students become engaged in a high level of cognitive complexity.</p>
<p><b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i></p>	<p>At least one enacted task engages students at a moderate or high level of cognitive complexity.</p> <p>Opportunity for complex thinking is limited in one or more of the following ways:</p> <ol style="list-style-type: none"> <li>Enactment of tasks that involve students at a high level of cognitive complexity comprise less than half of the lesson or are only provided to some students.</li> <li>The level of challenge is a clear mismatch for more than a few students.</li> <li>The teacher's interactions with students decrease the cognitive complexity of tasks that began as high-level task(s).</li> <li>Attention to a task typically ends when an answer is reached. There is little or no evidence of positioning tasks as a starting point for examining/extending mathematical ideas.</li> </ol>
<p><b>Low (0, 1)</b> <i>Contradicts principle</i></p>	<p>The cognitive complexity of enacted tasks is low, or the lesson engages students in little or no mathematical activity.</p> <p>The teacher assigns tasks that are routine and unambiguous with respect to exactly what the students are expected to do (e.g., provide answers without explanation; produce or recall isolated facts* from memory; identify or retrieve information from a graph, table, or figure; reproduce prescribed procedures without connections to concepts/underlying reasoning). Although it may be permitted, students are not expected to develop original methods or solutions to problems. (1)</p> <p>The teacher's interactions with students decrease the cognitive complexity of tasks that began as moderate-level task(s). (1)</p> <p>The teacher does not assign any mathematics tasks for students to complete independent of the teacher. Student engagement with mathematics problems may be limited to following explicit, step-by-step instructions for completing a task or review of homework or tests in order to score the responses as correct/incorrect without additional analysis or mathematical work. (0)</p> <p>*Facts may involve definitions, rules, terms, formulae, etc.</p>

## Principle 2: The teacher attends to students' mathematical thinking.

<b>Attend (ATND)</b> The teacher attends to the details in students' mathematical thinking processes.	
<b>High (4, 5)</b> <i>Strong to expert implementation of principle</i>	<p>The teacher attends to student thinking processes throughout the lesson by observing students and probing the details of students' strategies for solving problems.</p> <p>All of the following should be present:</p> <ol style="list-style-type: none"><li>The teacher appears physically and mentally attentive to varied aspects of individual students' mathematical thinking communicated through written artifacts, verbalizations, gestures, etc.</li><li>The teacher's probing questions are clearly tailored to the mathematical ideas shared by individual students.</li><li>The teacher probes student thinking in a way that minimizes assumptions and avoids asserting their own (different) thinking on the student.</li><li>The teacher asks follow-up questions to individuals that maintain focus on eliciting evidence of their mathematical thinking and understanding.</li></ol>
<b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i>	<p>The teacher sometimes attends to student thinking processes by observing students and probing details of strategies used to solve problems.</p> <p>The High rating is not warranted for one or more reasons:</p> <ol style="list-style-type: none"><li>The teacher's focus on students' thinking processes comprises a small portion of the time in the lesson or is limited to a small subset of students in the class.</li><li>The teacher does not always appear physically and mentally attentive as students share their individual mathematical thinking.</li><li>The teacher's attention to students' thinking processes is constrained to probing details and understanding of a procedure prescribed by the teacher/textbook.</li><li>Major assumptions are evident in the teacher's interaction with individual students and/or the teacher asserts their own (different) thinking on the student.</li><li>The teacher rarely asks follow up questions that are tailored to students' individual ways of thinking (e.g., questions are formulaic or leading).</li></ol>
<b>Low (0, 1)</b> <i>Contradicts principle</i>	<p>The teacher demonstrates a lack of interest or pays minimal attention to students' mathematical thinking processes.</p> <p>The teacher attends to students' ways of approaching problems primarily to check whether they are arriving at correct answers or using a prescribed procedure correctly. The teacher rarely or never asks questions that probe students' thinking or understanding. (1)</p> <p>The teacher rarely or never observes or monitors mathematical aspects of students' work while they are engaged in the process of solving a problem and does not ask students to explain their thinking later. (0)</p>

### Principle 3: Students engage with their peers’ ideas about mathematics.

#### Teacher Support for Peer Interaction (TSPI)

The teacher creates opportunities and provides support for students to interact with each other’s mathematical perspectives and ideas in order to advance their individual and collective understanding of mathematics.

*Students’ mathematical perspectives and ideas include elaboration of students’ own strategies as well as their conjectures, explanations, justifications, analytical comments and questions that aim to make sense of strategies introduced by others. Opportunities for students to interact with the ideas of peers may occur through direct student-to-student interaction or indirectly through teacher-mediated discussion.*

#### High (4, 5)

*Strong to expert implementation of principle*

The teacher creates ample opportunity and provides support for peer interactions in which students are expected to work to understand each other’s mathematical perspectives and collaborate to make sense of mathematical ideas. This may occur through partner work, small-groups, and/or teacher-facilitated discussion.

The teacher does ALL of the following:

- Consistently promotes/enforces expectation of mutually respectful interaction (e.g., by encouraging students to attend by listening and looking, coaching students how to respond to peers respectfully) or class culture provides strong evidence of previously established norms.
- Positions peers and their ideas as important resources for learning, including when students encounter difficulty.\*
- Invites/directs students to explain, add on to, and/or ask questions about other students’ mathematical ideas (e.g., by adding details, challenging with reasons, comparing, analyzing, justifying).

\*To score High, teachers do not need to *always* respond to difficulty by orienting students to their peers. Rather this indicator aims to detect some intentional positioning.

#### Medium (2, 3)

*Weak to moderate implementation of principle*

The teacher creates some opportunities for students to listen to, make sense of, and learn from each other’s mathematical ideas, but opportunities for equitable and substantive peer interaction are constrained.

The High rating is not warranted for one or more of the following reasons:

- Instances in which the teacher actively encourages students to attend to or interact with their peer’s mathematical ideas have limited prominence. (e.g., the teacher facilitates a brief public-sharing time after students have worked independently—and without peer interaction—for the majority of the lesson; partner work is brief or involves little or no substantive interaction).
- The teacher dominates discourse focused on students’ mathematical ideas, thus limiting students’ opportunities to explain their own ideas to peers (e.g., teacher does most of the sharing of student strategies/ideas).
- The teacher inconsistently enforces expectations of respectful peer interaction.
- The teacher positions himself as the only person from whom to seek help.
- When students’ mathematical strategies/ideas are shared, the teacher usually promotes passive or superficial response from other students rather than substantive response (e.g., ‘Show and Tell’ public sharing structure emphasizing listening, thumbs-up/down, choral response, brief turn-and-talk; partner work that involves listening to peers’ ideas but minimal responding).

#### Low (0, 1)

*Contradicts principle*

The teacher creates little or no opportunity for students to interact in ways that enable them to learn from each other’s mathematical ideas.

The teacher may invite students to demonstrate the solution to a problem or provide contributions that are brief and involve producing specific information, such as an answer to a problem or a step in a procedure that has been introduced by the teacher or textbook; student interaction may be limited to grading each other’s work according to an answer key (e.g., scoring a timed test). (1)

The teacher does not create opportunities for students to be exposed to each other’s mathematical ideas or solutions (i.e. process, answers, written work)—and may even discourage it. (0)



**Principle 4: The teacher supports students’ understanding and meaningful use of the language of mathematics.**

<b>Peer Interaction (PRI)</b> <b>Students interact with each other to support the advancement of their individual and collective understanding of mathematics.</b>	
<p><b>High (4, 5)</b> <i>Strong to expert implementation of principle</i></p>	<p>Students listen, attend, and actively respond to each other’s mathematical ideas in substantive ways, and this peer interaction is a prominent feature of the lesson. Response to peers’ ideas can take place in collaborative groups and/or teacher-facilitated discussion. It can be student-initiated or prompted by the teacher.</p> <p>Students may demonstrate substantive interaction with a peer’s idea by:</p> <ol style="list-style-type: none"> <li>a. Restating or explaining the details of a peer’s idea</li> <li>b. Making a conjecture about a peer’s strategy (e.g., I think Ann did that, because...)</li> <li>c. Referencing or elaborating on someone else’s idea</li> <li>d. Asking questions about the details in their peer’s strategy</li> <li>e. Explaining why they agree or disagree with a peer’s idea</li> <li>f. Comparing or contrasting their ideas with those of their classmate(s)</li> </ol>
<p><b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i></p>	<p>Students interact with each other’s mathematical ideas in primarily passive or superficial ways rather than active and substantive ways.</p> <p>The High rating is not warranted for one or more of the following reasons:</p> <ol style="list-style-type: none"> <li>a. Peer interaction in which students attend to or interact with each other’s ideas comprises a small portion of the lesson or is only experienced by a small subset of students (e.g., a table group).</li> <li>b. Students mostly listen passively to their peer’s strategies or mathematical ideas or interact with their peers’ ideas in superficial ways (e.g., use of hand signals—when prompted by the teacher—to indicate agreement/disagreement with a peer’s idea or approval/disapproval of their solution to a mathematics problem).</li> <li>c. When directed to engage with other students mathematical ideas in substantive ways, many students are inattentive or unresponsive.</li> </ol>
<p><b>Low (0, 1)</b> <i>Contradicts principle</i></p>	<p>Students exhibit little or no interaction with their peer’s mathematical ideas. Instruction primarily engages students with their own ideas or with those of the teacher or textbook.</p> <p>Students are exposed to each other’s brief answers but have little or no interaction with each other’s mathematical ideas. Most student contributions—in public parts of the lesson and to collaborative groups/partner-work—are brief and involve repeating specific information, such as an answer to a problem or steps in a procedure that has been introduced by the teacher or textbook. (1)</p> <p>Students have little or no opportunity to share their mathematics-related answers or ideas with peers, directly or indirectly via teacher-mediation. (0)</p>

**Principle 4: The teacher supports students’ understanding and meaningful use of the language of mathematics.**

**Express-Elaborate (EE)**

**The teacher actively supports students with developing and refining their abilities to express and elaborate mathematical ideas.**

*Expression/elaboration can occur in different modalities, including spoken words, written symbols or pictures, demonstrations with manipulatives, etc.*

<p><b>High (4, 5)</b> <i>Strong to expert implementation of principle</i></p>	<p>The teacher creates abundant opportunities during the lesson for students to practice expressing mathematical ideas with elaboration.</p> <p>All of the following are present:</p> <ol style="list-style-type: none"> <li>The teacher creates opportunity and provides sufficient time for many or all students to practice expressing their ideas with elaboration during the lesson.</li> <li>The teacher provides support for students to improve the clarity and precision (i.e., lack of ambiguity) of their expressions and elaborations (e.g., by using revoicing strategies to help students to express their ideas clearly and unambiguously, by pressing for additional explanation, by encouraging students to add labels to a picture representation of a solution).</li> </ol>
<p><b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i></p>	<p>The teacher creates some opportunities for students to express mathematical ideas with elaboration.</p> <p>The High rating is not warranted for one or more reasons:</p> <ol style="list-style-type: none"> <li>Opportunities for students to express their ideas with elaboration comprises a small portion of the lesson or are limited to a small subset of students in the class.</li> <li>Students are invited or directed to express mathematical ideas with elaboration, but sufficient time is not provided (e.g., the teacher rushes a student’s verbal explanation or cuts in and speaks for them; the teacher cuts short opportunity for written expression before many students are finished).</li> <li>When students do express their ideas, the teacher rarely or never presses for additional explanation or elaboration.</li> <li>The teacher rarely or never provides feedback to students to help them improve the clarity, completeness, or precision of their expressions.</li> </ol>
<p><b>Low (0, 1)</b> <i>Contradicts principle</i></p>	<p>The teacher rarely or never creates opportunities for students to express their mathematical ideas with elaboration.</p> <p>Questions directed at students mostly elicit brief responses or answers (e.g., one-word replies, filling in a blank on a worksheet). I-R-E discourse patterns predominate the interactive component of classroom discourse. (1)</p> <p>Opportunities for students to express mathematical ideas are focused primarily on recounting procedures taught by the teacher or textbook (but not explaining or interpreting them). (1)</p> <p>The teacher never, or almost never, invites students to express their mathematical ideas during the lesson, verbally or otherwise. (0)</p>

**Principle 4: The teacher supports students’ understanding and meaningful use of the language of mathematics.**

**Connecting Representations (CR)**

**The teacher facilitates opportunities for students to examine conceptual connections among different external representations of mathematical concepts to support the advancement of their understanding and use of mathematics content and language.**

*Representations may be internal (mental) or external; in this rubric, we focus on representations that comprise external manifestations of mathematical concepts. These external representations may occur in visual, verbal, contextual, physical, or symbolic modalities. External representations may be used as tools to support thinking in the midst of solving problems, or they may be used to support explanation and reasoning after a problem has been solved. External representations may be used to introduce or review the meaning of mathematical concepts, notation, or terminology. External representations are not required to conform to the standard/formal lexicon in the broader mathematics community. The source or creator (e.g., teacher, student, textbook) of the representation does not affect the rating on this rubric.*

<p><b>High (4, 5)</b> <i>Strong to expert implementation of principle</i></p>	<p>The teacher facilitates opportunities for students to notice and verbalize connections among external representations to advance students’ understanding and meaningful use of the language of mathematics and the mathematical concepts they represent.</p> <p>The teacher does all of the following:</p> <ol style="list-style-type: none"> <li>In public parts of a lesson, ensures nonverbal representations of mathematical ideas are consistently accessible and used to anchor discussion.</li> <li>Presses many or all students to compare the underlying mathematical meaning of two or more external representations (visual, contextual, physical, or symbolic).</li> </ol> <p>* Indicator b may be observed during public parts of the lesson, in the context of one-on-one interactions between teacher and student, or through prompts on a written assignment.</p>
<p><b>Medium (2, 3)</b> <i>Weak to moderate implementation of principle</i></p>	<p>The teacher creates some opportunities for students to examine conceptual connections among multiple external representations of a mathematical concept, but the opportunities are predominantly passive or constrained in some way.</p> <p>The High rating is not warranted for one or more of the following reasons:</p> <ol style="list-style-type: none"> <li>Nonverbal representations do not consistently serve as an anchor or common touchstone for public examination of connections (e.g., display of representation is difficult to see, is too brief to support examination, or limited examination of connections occurs).</li> <li>Instructional focus on conceptual connections among multiple representations comprises a small portion of the lesson or is only experienced by a subset of students.</li> <li>The teacher provides detailed and mathematically relevant comparisons of two or more external representations but rarely or never presses students to explain the connections.</li> <li>The teacher prompts students to compare two or more external representations, but this is done in primarily superficial or unsustained ways.</li> <li>The teacher or students explain the meaning (orally or in writing) of one representation at a time, but they rarely or never compare or contrast mathematically meaningful aspects of two or more representations.</li> </ol>
<p><b>Low (0, 1)</b> <i>Contradicts principle</i></p>	<p>The teacher neglects to identify or direct students’ attention to conceptual connections among different external representations or the mathematics concepts they represent.</p> <p>Students observe or create multiple representations of mathematical ideas with the same underlying structure, but there is little or no explicit attempt by the teacher or students to compare, contrast, or interpret the underlying meaning of the representations. (1)</p> <p>References to external representations and their use focus on superficial features (e.g., naming the type of representation) or demonstration of procedures and neglect to discuss their structural equivalence or connections to the concepts they represent. (1)</p> <p>There is no opportunity for members of the class to examine or discuss connections among representations of a mathematics concept. (0)</p>

## Scoring Guidance

Classroom instruction should be scored High if it is consistent with the given CGI principle.

It should be scored Medium if some elements of the CGI principle are present.

It should be scored Low if there are elements present that contradict the CGI principle.

### **Within High:**

- Score 5 (Expert) if there is consistently high expert-level performance in relation to the main description and indicators listed. This score is not expected to be assigned frequently and should be reserved for truly exemplary implementation of the rubric's focus.
- Score 4 (Strong) if evidence of all indicators required by the High criteria is present but not consistently strong (or representative of expert-level performance) AND the disqualifying criteria in the Medium level do not apply.

### **Within Medium:**

- Score 3 (Moderate) if the performance has some characteristics in common with a High rating, but some aspect of what is observed does not meet criteria specified for a High rating. This score is usually appropriate if only one indicator in Medium applies.
- Score 2 (Weak) if the performance has few or no characteristics in common with a High rating, but the criteria for a Low rating also does not describe what has been observed. Usually this score is appropriate if two or more indicators in the Medium level apply.
- Occasionally, the listed indicators in Medium on a given rubric do not seem a perfect fit for what has been observed, but the performance is reflective of the main statement in Medium. When this occurs, observers should confirm that the criteria in High are not satisfied and the criteria in Low do not describe what has been observed. Then a score of 3 or 2 should be assigned based on a holistic judgment of whether the performance is represents a Moderate (3) or Weak (2) implementation of the construct of focus.

### **Within Low:**

- Scores 1 and 0 both indicate performance contradicts the principle, with 0 representing a greater degree of contradiction.
- The specific criteria for 1 and 0 are provided within each rubric.