CONTEXTUAL MEANINGS OF THE EQUALS SIGN AS CONCEPTUAL BLENDS

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Students today may more readily recognize the symbol # as the hashtag used on Twitter and other social-media platforms than as a symbol for weight, musical notation for a sharp note, the pound key on a telephone, or as shorthand for *number*. Digital media add to the many streams of symbol-rich information children experience, so *recognizing* can be seen as an important and active interpretive process affected by experience, awareness, and sense of ownership. In social media, mathematical language and symbols are being appropriated in ways that stretch their meaning and appear to create new combinations. This process, which children experience simultaneously with formal instruction, presents mathematics educators and researchers with both challenges and opportunities.

Signs and texts are subject to reinterpretation and contextualized translation into new contexts. Mathematics education research faces the challenge of embracing multiliterate mathematical thinking to support students' adaptive production and consumption of mathematical texts. We argue that the blending of mathematical symbols, expressions, and thinking from formal to informal uses and from school to non-school contexts represents a normal multiliterate feature of 21st century life, a process that children can become aware of and use as they expand their communicative repertoires both in and out of school. We are interested in how a ubiquitous symbol—the equals sign—is appropriated and used contextually in advertising and social media.

In cross-cultural contexts, 'distance' between disciplinary norms for mathematical communication can clash with local linguistic and cultural histories (Berry, 1985, p. 19). And ambiguity between everyday and specialized meanings of mathematical words and symbols can be a source of mathematical 'misinformation' (Durkin & Shire, 1991, p. 73). Diverse appropriations of signs *within mathematics discourses* point to deep polysemy, so strong 'awareness' of context may be seen as an antidote to misinformation (Zazkis, 1998, p. 30). These issues can be viewed as central to young learners' experience, as 'new' contexts suggest meanings that contrast with 'familiar' ones (Mamolo, 2010).

Views of the equals sign

Progression from simpler to more complex mathematical thinking is a vital concern for educators and researchers. Developing awareness in research of the need among teachers and students to manage ambiguity contrasts to some degree with research and teaching goals having to do with the equals sign.

An operational view of the equals sign among school children (*e.g.,* Knuth, Stephens, McNeil & Alibali, 2006) is a problem from the standpoint of algebra learning. Framing the problem in terms of future roadblocks to high-school mathematics achievement, equals sign interventions have introduced typologies and made instructional recommendations focussed on reasoning relationally. Some seek to shape the discursive landscape in which children encounter the equals sign, for instance, by adopting the language of sameness (*e.g.,* reading the equals sign as 'is the same as') (Saenz-Ludlow & Walgamuth, 1998), using true-false questions (Molina & Ambrose, 2006), or introducing the metaphor of the balance scale or seesaw (Mann, 2004).

Efforts to steer children's understanding of the equals sign through the use of particular semiotic tools depend on recognition that differing views of the sign may coexist in the mind of the child. In other words, children's understandings of the equals sign are negotiated, not simply sequenced developmentally. Light (1980) identified six meanings of the equals sign within mathematics; Baroody and Ginsburg (1983) raised the possibility that multiple views coexist; and Jones and Pratt (2012) found that some students used two distinct meanings of the equals sign, a relational meaning and a substituting meaning, to create arithmetic puzzles. Documented cases of children flexibly recruiting different meanings to successfully complete tasks point to the importance of the child as a negotiator of meaning in relation to signs and context.

Our analysis explores how people use and see the equals sign in contexts outside of mathematics, especially social media and advertising. We consider the variety of meanings that these ways of using the equals sign evoke as evidence of the multiliterate environments that people navigate on a daily basis. We propose that these media-saturated environments call for pedagogical approaches to children's mathematical understanding that take into account the linguistic and cultural contexts in which equals signs are used. This perspective has implications for efforts to assess human understanding of words and symbols, because understanding and language use occur in a variety of contexts.

Conceptual blending

We frame our analysis of contextual meanings of the equals sign in terms of linguistic pragmatics, specifically how conceptual blending (Fauconnier & Turner, 2002) supports sign users' interpretation of abstract symbolic meaning (Peirce, 1903). Conceptual blending is a process by which elements from different source domains, or input spaces, are comingled to create a unique, blended space, resulting in both gains and losses in terms of what people appear to understand (Zandieh, Roh & Knapp, 2014). A blend involves a

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combination of elements from each source domain or hybrid elements influenced by both source domains. Conceptual blends are ubiquitous and may be found in everyday language, politics, literature, technology, mathematics, and advertising. In mathematics education research, conceptual blending has been used to investigate students' mathematical reasoning (Zandieh *et al.,* 2014), as well as preservice teachers' feelings about mathematics (Zazkis, 2015). Here, we use it to illustrate the existence of divergent meaning-making pathways.

We diagram a variety of conceptual blends we created as we *recognized* the equals sign in social media and advertisements. Figure 1 shows how we illustrate potential source domains, corresponding elements, and a blended space [1]. Within the circle representing Source Domain 1, we present phenomena belonging to that source domain relevant to the blend. We then specifically identify the elements that are involved in our blend. We do likewise for our Source Domain 2, and we use dotted line segments to illustrate the fusion of elements into the blended space. To further clarify our conceptualization of a given blend, we also describe the categorical nature of each element in the source domains and then the categorical nature of the elements in the blended space. The latter may derive from one or the other of the source domains or involve a fusion of meanings deriving from the two domains.

Of special importance to our inquiry is the pragmatic linguistic principle that symbols acquire their potential to mean through a variety of cultural processes, with the result that symbols can have competing meanings by virtue of their differing histories (Peirce, 1903). The construction of relations between symbolic signs and their objects, in this case a concept or perhaps multiple concepts, sets them apart from other signs, resulting in layers of potential meaning for symbols as they are imported into new contexts.

Ways of using the equals sign in advertising and social media

Below, we consider ways in which the equals sign is being used in advertising and social media, sorted into categories based on the sorts of conceptual blends we created as we read the expressions. We analyze several examples by framing them as conceptual blends that bring together distinct domains. The categories presented in this article exhibit the diversity of conceptual blends, not a comprehensive or authoritative sorting of possible blends. Nor do we attempt to quantify the relative frequencies of these ways of using the equals sign in the world. Our purpose is to demonstrate the context-dependent variability of the equals sign's meaning and to consider the implications of conceptual blending for contemporary mathematics education. We describe the complex thinking involved in these blends as manifestations of often invisible sense-making processes children and adults readily perform. This inter-psychological work necessitates adaptive understanding of formal mathematical language appropriated in unpredictable but intelligible ways.

Plus Result

Figure 2 is an advertisement warning against drunk driving, especially the driving of motorcycles. The statement in this case is 'DRINK + RIDE = LOSE' with the further dramatic specification that 'FREEDOM!' is what will be lost [2]. Plus Result statements specify two or more inputs or conditions that together yield some result or consequence. Another example is 'Sweat + Sacrifice = Success'. In such statements, we infer the meaning for the equals sign as 'yields' or 'results in' or 'causes'. Plus Result statements are one-directional and meant to be read from left to right. Playful manipulations (based on the rules of arithmetic) yield entertaining but nonsensical statements, such as 'LOSE – DRINK = RIDE'. Plus Result statements are not relational in

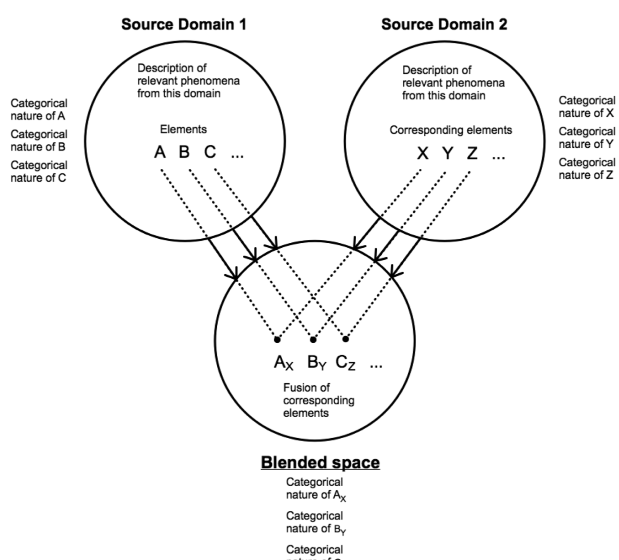

Figure 1. Generic illustration of a conceptual blend.

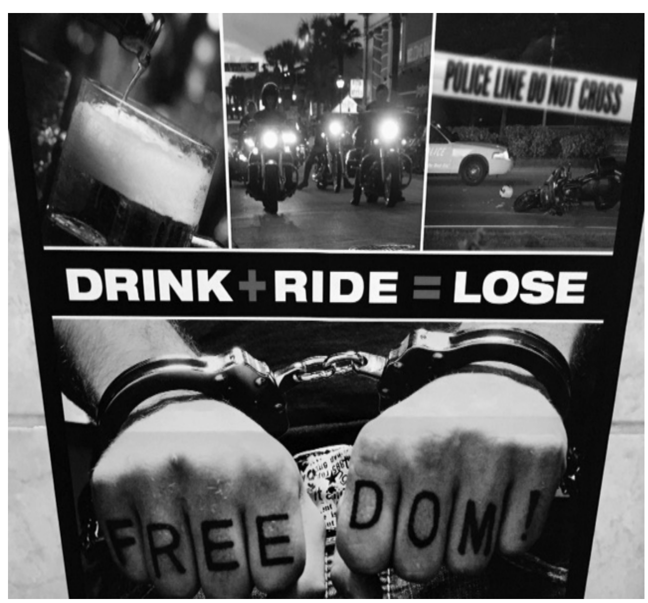

Figure 2. Advertisement warning against drunk driving.

nature, and so the rules for manipulating algebraic equations may not apply.

We conceive of 'DRINK + RIDE = LOSE' as existing in a blended space (Figure 3). We describe one of the source domains for this blend as the domain of cause and effect, which consists of real-world activities and their consequences. In this domain, the combination of the activities of drinking and driving may yield negative consequences, including being arrested. The second source domain is school mathematics, especially elementary arithmetic. Within this domain, equations often take an operational form in which the left-hand side presents a computation to be performed and the right-hand side is the place for the answer. In the blended space, the meanings of parallel components are fused: Activities are treated as addends, and addition (+) is used as a conjunction (*and*). Rather than calling for a computation (addition), the equals sign (=) calls for the consequence of combining two activities. Likewise, rather than the right-hand side giving the sum, it presents the consequence or result. In this blended space, the statement 'DRINK + RIDE = LOSE' becomes sensible and may be understood as intended. The ability to meaningfully interpret the statement requires viewers to construct a blended space like the one in Figure 3. Viewers' familiarity with the relevant source domains—including the operational meaning of the equals sign—makes this blend possible.

Simple Result

The Simple Result form relates to the Plus Result form described above. In the *Simple Result* form, the statement conveys a causal relationship or logical implication without using + or another operator, as in 'Less resentment = more gratitude'. In these statements, an implication arrow (⇒) could sensibly be substituted for the equals sign. Ally Bank recently ran an advertising campaign in which the Simple Result form was central. Each of these commercials opens

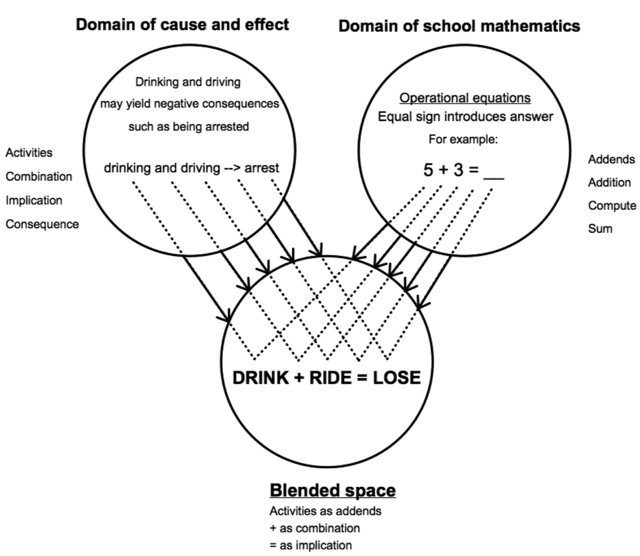

Figure 3. A blended space in which 'DRINK + RIDE = LOSE' becomes sensible.

Figure 4. Images from an Ally Bank commercial.

with the following declaration: 'At Ally Bank, NO BRANCHES = GREAT RATES'. Each commercial then goes on to draw an analogy to another Simple Result scenario. For example, one ad takes place on a golf course and states, 'PLAYING BOSS = BOSS WINS' (Figure 4). In each Simple Result statement, there is a clear cause-and-effect, or entailment, relationship between the entity, situation, or phenomenon on the left-hand side of the equals sign and the one on the right.

'PLAYING BOSS = BOSS WINS' shares some features with 'DRINK + RIDE = LOSE' (such as its one-directional nature), but it exists in a unique blended space (Figure 5). Again, this blend draws on the source domain of school mathematics, particularly the operational meaning of the equals sign. In this case, that meaning is more general. The mapping does not involve a particular form of mathematical equation, such as A + B = C, but instead makes use of a more general template for equations in which a problem appears on the left-hand side and its answer belongs on the right-hand side. The other source domain for this blend is that of office politics. In this domain, it may be advisable to let one's boss win if engaged in a competitive activity such as golf. Given this advice, playing golf with one's boss implies that the boss will win. Thus, in the blended space, playing golf with one's boss takes on the role of the problem or situation, the equals sign (=) introduces a consequence or implication, and the result or answer is that the boss wins the game. In this way, 'PLAYING BOSS = BOSS WINS' succinctly conveys advice (or makes a joke) about office politics by borrowing the operational meaning of the equals sign from school mathematics.

Definitional and Descriptive forms

One tweet that we came across read, "Low quality = no story, bland, or content without emotional connections".

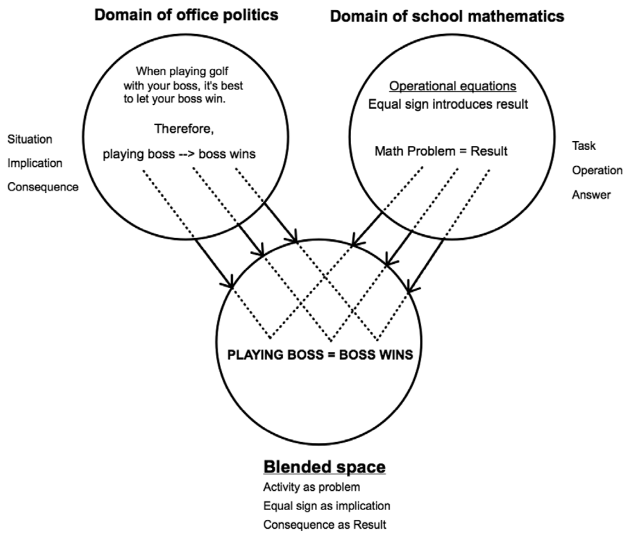

Figure 5. A blended space in which 'PLAYING BOSS = BOSS WINS' becomes sensible.

This statement seems to convey the author's definition of low quality, presumably in the context of a book, movie, or the like. In this case, the left-hand side introduces the subject and the right-hand side provides a definition. This form is closely related to the Descriptive form. *Descriptive* statements present a qualitative description as in 'Spring break = me re-watching every show on Netflix'. This is not a general definition of spring break, as it is specific to the author. It offers a descriptive account of what spring break looks like in the author's life. In these statements, we would infer a meaning for the equals sign along the lines of 'is' or 'means'—in this case, that meaning is personal.

Figure 6 presents the blended space in which 'Spring break = me re-watching every show on Netflix' becomes sensible to us. On the left is the source domain of seasonal behavioral patterns. At different times of year, people behave in different ways. For this person, spring break is a time that consists of sitting at home and re-watching shows on Netflix. On the right is the source domain of arithmetic expressions. In this domain, there are short and long forms for numbers. The longer, expanded forms unpack the number in ways that emphasize specific meanings or relationships. For example, 10 may be decomposed additively as $9 + 1$ or $8 + 2$, *etc*. Alternatively, 10 may be decomposed multiplicatively as 5×2 or 10×1. In expressions such as $10 = 9 + 1$, the equals sign conveys equivalence. Specifically, it may introduce one of many possible expressions that are equivalent to the number or expression on the left hand side. In the blended space, 'Spring break = me re-watching every show on Netflix' conveys the idea that spring break may mean different things in the lives of different people; for the individual making the statement, that time of year means spending many hours watching shows on Netflix.

Evaluative, State, Report, and Value forms

The Evaluative form is related to the Descriptive form; however, *Evaluative* statements present a particular value judgment, not merely a description. One example is 'Today = Epic'. This statement provides an evaluation of the quality of the day. In this case, that evaluation is very positive. It would contrast with a statement such as 'Day = fail' that conveys a definitively negative evaluation. Another close cousin to the Descriptive form is the *State* form, which specifies a current state of affairs, as in 'hair = mess'. Such statements describe a temporary state. An evaluative aspect may be implicit—in this case, messy hair has a negative connotation—but evaluation is not necessary.

The statement 'hair = mess' conjures up a relatively simple blended space that is distinct from those described previously (Figure 7). This blend draws on the source domain of physical appearance, including the normative and evaluative features of that domain. In particular, people are often expected to appear well groomed and presentable. In this domain, messy hair is generally regarded as an undesirable and somewhat embarrassing state of affairs. The use of the equals sign in 'hair = mess' thus specifies the current state of an entity whose state tends to vary from day to day. This meaning is analogous to that of a variable. In the domain of algebra, a variable such as x may take on different numerical values. In this context, the statement $x = 5$ does not indicate that x is defined to be 5, but rather that 5 is its current value or the value under discussion (as in, 'Evaluate $f(x)$ when $x = 5$'). In the blended space, then, hair behaves like a variable, the current state of which is 'mess'. The statement 'hair = mess' carries with it all the connotations from the domain of physical appearance, while appropriating a particular way of using the equals sign from the domain of algebra.

Also closely related to the Evaluative and State forms is the *Report* form which provides a quantitative summary of recent events, such as 'Last 24 hours = 1 follower and 3 unfollowers'. This statement means that in the last 24 hours, the person's Twitter account gained one follower and lost three followers. Again, there may be evaluative associations

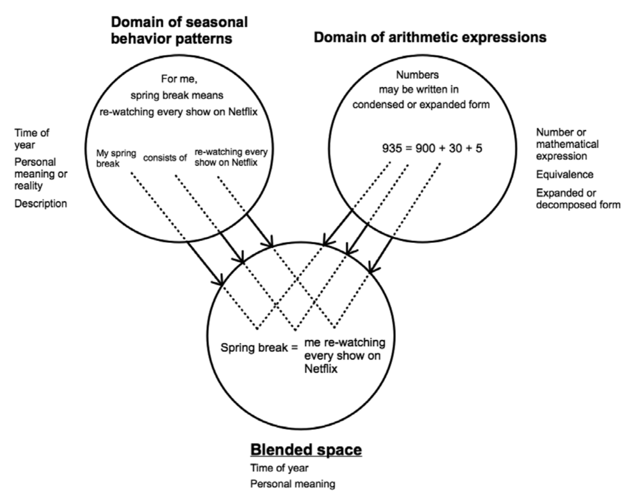

Figure 6. Blended space of 'Spring break = me re-watching every show on Netflix'.

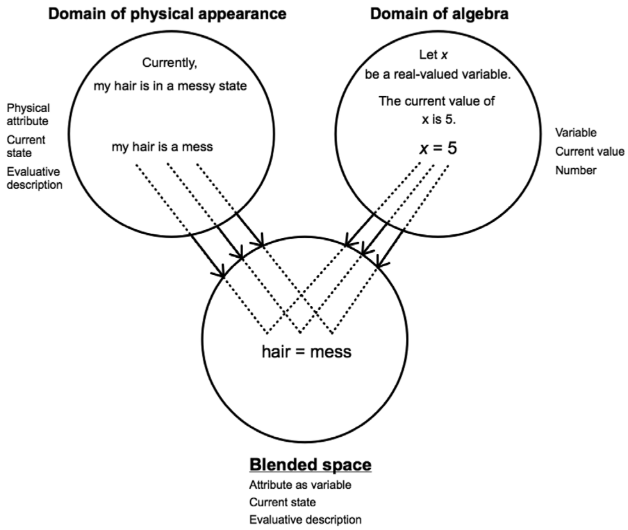

Figure 7. Blended space in which 'hair = mess' becomes sensible.

to such reports. In this case, losing followers is generally regarded as undesirable, so to lose more followers than one gains in a 24-hour period likely constitutes a bad day in the social-media domain. The related Value form simply conveys the *value* or exchange rate for something, as in '1 retweet = 1 vote'.

Discussion

These categories and examples illustrate various ways in which the equals sign is being used in advertising and social media. It is clear that people must invoke multiple, contextually appropriate meanings of the equals sign in order to interpret such statements sensibly. The similarities and differences among these blends—especially their mathematical source domains—illustrate the nuanced differences in contextual meaning that people readily negotiate to sensibly interpret various statements involving the equals sign. Furthermore, common ways of using the equals sign, especially the Plus Result and Simple Result forms, draw upon the operational interpretation of the equals sign, and thus might be seen as potentially having a negative influence on students' learning of mathematics. Conceptual blending can result in loss of meaning in addition to the production of new meanings. We argue that the appropriate teaching response is to cultivate ownership among students—not only of the relational meaning of the equals sign, but of the meaning-making process itself—so that gains and losses occurring as a result of conceptual blending reflect children's active participation in formal and informal mathematical meaning-making.

The variety, familiarity, and intelligibility of mathematical expressions point to their stability as cultural reference points (Hutchins, 2005). This stability supports the expansion of conceptual blends as communication goals, technologies, and contexts change, but the possibility of contraction or reinforcement of problematic conceptions requires attention. This relation between contemporary communication and mathematical contexts points to a challenge—that popular cultural references to the equals sign reinforce meanings that may interfere with algebraic thinking. However, the same cultural salience presents mathematics educators with an opportunity to help learners develop critical awareness of the context-bound meanings of the equals sign.

Highly influential and successful programs in mathematics education have been founded upon the premise that instruction should begin with meaningful situations that relate to students' prior experiences and cultures. Students can learn mathematics as they learn to mathematize tasks that are posed in familiar contexts. However, the cases and categories we have presented suggest that in the context of popular culture, mathematical expressions themselves function as culturally familiar contexts for *other* types of communication.

Implications

Durkin and Shire (1991) state that critical awareness can be a real solution to the problem of ambiguity between informal and formal uses of mathematical language. They suggest we embrace, not shy away from, opportunities to differentiate formal from informal contextual meanings, productively attending to children's complex combination of language and experience as a basis for mathematics problem solving.

Zazkis (1998) applies Durkin and Shire's recommendations to within-register ambiguity in elementary mathematics. She challenges teachers and researchers to frame conflicting mathematical meanings as teaching opportunities geared toward helping students find their place within specific subdisciplinary contexts. Extending the investigation of polysemy to new media which draw upon specialized mathematical source domains leads to similar conclusions: Just as ambiguity in everyday language creates opportunities to understand mathematical relations in specialized ways, and within-register ambiguity supports students' development of 'awareness' of specific subdomains of mathematical contexts, so conceptual blends drawing on mathematical meanings create potentially generative conflicts among meanings of the equals sign.

For instance, some of the blends that we illustrated helped us recognize symmetrical uses of the equals sign, while others appeared asymmetrical. Critical awareness could mean using metaphors like symmetry versus asymmetry, or perhaps a playground see-saw versus a slide, to help children notice the conceptual blending they are doing as they navigate mathematical expressions in everyday and formal contexts. It seems useful, then, to acknowledge cases in which a particular metaphor fits the context and cases in which it does not.

As it is, mathematics instruction suffers from being distanced from real-life application. This is especially the case at the secondary level, where many students and teachers struggle with the question, "Where will I ever use this?" In the mass adoption of the equals sign as a linguistic form in advertising and social media, there is an opportunity to acknowledge and leverage the crossover between school mathematics and the outside world. Ignoring or dismissing these crossovers renders school mathematics more disconnected from students' lives outside of school.

Expressions like 'hair = mess' play with the look and feel of mathematics. If these constructions are intelligible to readers, then those readers necessarily draw upon resources beyond formal mathematics to produce contextual meanings of the equals sign. The question for the field of mathematics education is whether our work helps students to better navigate the variety of contextual meanings of words and symbols such as the equals sign. In other words, do we miss such opportunities out of fear of confusing students or interfering with a clean progression from operational to relational? There has been great concern over how to get students to adopt and follow the relational meaning, but we argue that achieving this goal should include holistically supporting their efforts to make sense of the equals sign in contextually appropriate ways.

Mathematics teachers can open a conversation with students by inviting them to notice how the equals sign is used in different contexts and to ask themselves what it means in each context. Do patterns appear? Can these examples be sorted or related to formal mathematical expressions? Such activity would foster a valuable meta-cognitive habit that would serve students well both in and out of school. Common approaches to teaching equation solving—such as having students draw a vertical line segment that extends down from the equals sign, separating its two sides—seem to be concerned with restricting students' thinking and activity in ways that will force them to perform correctly, as opposed to encouraging critical awareness and empowering students to make important distinctions. It may be that many students feel a sense of ownership of the operational but not the relational meaning. We would advocate approaches that encourage students to take ownership of a variety of meanings of the equals sign.

Notes

[1] Note that in specifying corresponding elements, we are not claiming that those elements are otherwise analogous. Conceptual blends are distinct from analogies which do not involve a blended space. The corresponding elements that we identify correspond for the purposes of the blend but may otherwise have no apparent analogical relationship.

[2] We do not advocate using this particular example as the basis of discussion with children, but it illustrates a type of blended meaning of the equals sign we think is ubiquitous.

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A conversation I had a long time ago comes into my mind. One of our writers, a dear friend of mine, was complaining to me that he felt his education had been neglected in one important aspect, namely he did not know any mathematics. He felt this lack while working on his own ground, while writing. He still remembered the co-ordinate system from his school mathematics, and he had already used this in similes and imagery. He felt that there must be a great deal more such usable material in mathematics, and that his ability to express himself was all the poorer for his not being able to draw from this rich source.

— Rózsa Péter, *Playing With Infinity: Mathematical Explorations and Excursions*